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Engine Oil Inventory Management: Comparative Study of Lot-Sizing Methods and Managerial Decision Framework under Cashflow Constraints

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Abstract. *Small and medium-sized enterprises often need to maintain product availability while operating under limited working capital. In this context, a lot-sizing method that minimizes inventory-related costs may still generate purchasing expenditures that are concentrated in particular months and difficult to finance. This study evaluates five established lot-sizing methods namely Wagner-Whitin (WW), Lot-for-Lot (L4L), Economic Order Quantity (EOQ), Periodic Order Quantity (POQ), and Part-Period Balancing (PPB) using data from 13 high-turnover engine-oil SKUs and develops a liquidity-aware policy-selection framework. Order schedules and Total Inventory Cost (TIC) were generated using POM-QM, while cost differences were tested through repeated-measures analysis with Greenhouse-Geisser and Bonferroni adjustments. Monthly purchasing expenditures were then evaluated through violation frequency and cumulative excess under ceilings of IDR 40 million, IDR 50 million, and IDR 60 million. The lot-sizing method significantly affected TIC ($p < 0.001$). WW generated the lowest aggregate TIC, whereas L4L generated the highest TIC but the lowest cumulative excess across all scenarios. The cost-first and cash-first policies consistently selected WW and L4L, respectively. The balanced policy selected no feasible method at IDR 40 million, EOQ at IDR 50 million, and WW at IDR 60 million. These findings show that inventory-cost efficiency and purchasing-liquidity exposure require separate but integrated evaluation.*

Keywords: *Inventory planning, lot-sizing methods, total inventory cost, liquidity exposure, policy selection*

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Introduction

Motorcycle ownership in Indonesia has increased steadily, with data from Badan Pusat Statistik (BPS) indicating average annual growth of more than 5%. This expansion has increased demand for maintenance services and spare parts, with engine oil representing a major component of routine motorcycle maintenance. Records from IM Workshop during 2024 show that engine oil accounted for an average of 30.05% of total spare-parts demand, highlighting its dominant role, as presented in Figure 1. In December 2024, for example, brake pads represented only 3.57% of total demand, whereas engine oil accounted for 31.02%. This substantial difference indicates that maintaining engine-oil availability has considerable operational importance.

The Pareto analysis shows that 13 of the 65 engine-oil SKUs accounted for approximately 78.53% of total oil demand and were therefore selected as the focus of this study.

The concentration of demand among these items underscores their strategic importance: shortages may reduce customer satisfaction and service reliability, whereas excessive inventory ties up working capital. Prior studies reinforce this challenge. Panigrahi et al. (2024) and Kittisak (2023) highlight that small enterprises commonly face cash-flow limitations, restricted storage capacity, and inventory-fluctuation risks. Heizer et al. (2017) note that inventory can absorb a substantial proportion of working capital, making inventory control a critical managerial concern. Silver et al. (2016) similarly emphasize that structured ordering policies are necessary to control costs under financial pressure. Field observations at IM Workshop indicate that procurement decisions remain largely manual and intuitive, resulting in inefficient allocation across multiple items. Rompotis (2024) identifies cash-flow policy as an important determinant of working-capital efficiency, while Boardman et al. (2018) argue that decisions under financial constraints should consider the benefits generated by each unit of expenditure.

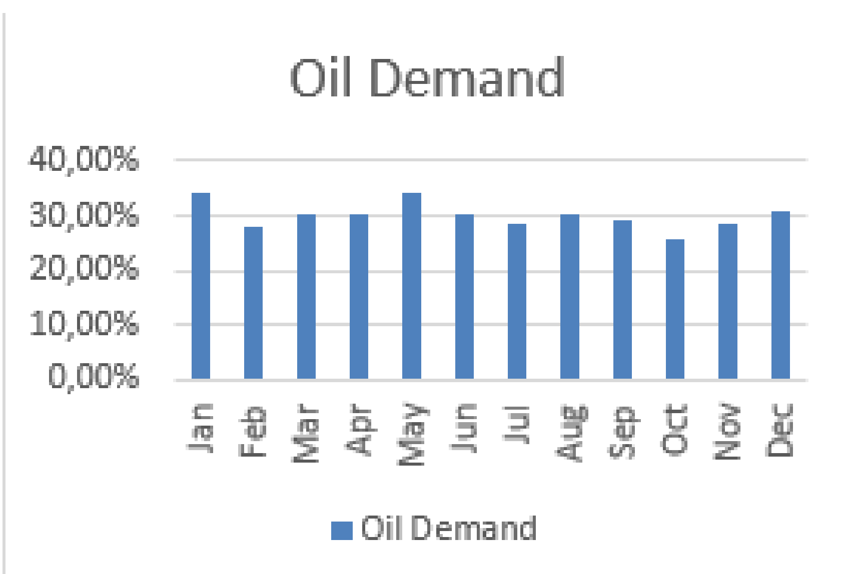

Figure 1.
Oil Demand Data in Relation to Total Spare Parts Demand in 2024

For the 13 high-turnover SKUs, the average monthly procurement expenditure was approximately IDR 44.3 million, ranging from IDR 36.1 million in February to IDR 51.8 million in December. A monthly purchasing ceiling of IDR 50 million was introduced as the base-case managerial benchmark, representing approximately 96.5% of the peak monthly procurement requirement. This benchmark was intended to accommodate purchasing requirements in most observed months while limiting liquidity exposure during periods of concentrated procurement. To examine whether the evaluation results depended on this single benchmark, the study also considered alternative ceilings of IDR 40 million and IDR 60 million, representing a 20% decrease and increase from the base case. These scenarios reflect tighter and more flexible working-capital conditions, respectively. This approach is consistent with the need to evaluate inventory efficiency under different levels of financial flexibility.

Classical lot-sizing methods, including Wagner-Whitin (WW), Lot-for-Lot (L4L), Economic Order Quantity (EOQ), Periodic Order Quantity (POQ), and Part-Period Balancing (PPB), offer different trade-offs between ordering frequency and inventory holding. Comparative applications of these methods commonly evaluate performance through Total Inventory Cost (TIC), defined in this study as the sum of ordering and holding costs. However, for small enterprises, a method that generates a low TIC may still create concentrated procurement requirements that are difficult to finance. Inventory-cost performance should therefore be evaluated separately from, but jointly with, monthly purchasing-expenditure profiles.

Recent studies at the interface of inventory management and finance have increasingly emphasized that inventory decisions should be evaluated together with cash-flow conditions. Katehakis et al. (2016) demonstrate that small and medium-sized enterprises must jointly manage inventory and cash decisions because ordering policies are influenced not only by demand and cost parameters, but also by available capital, borrowing capacity, and the firm's net worth. Chen and Rossi (2021) further show that, in cash-constrained inventory systems, available cash directly restricts the maximum quantity that can be ordered in each period, making cash availability a dynamic operational constraint rather than merely a financial outcome. In a multi-period and multi-product setting, Chen and Zhang (2024) also indicate that cash availability, delayed revenue collection, and financing mechanisms influence replenishment decisions under stochastic demand.

These studies provide an established theoretical basis that inventory planning under financial constraints requires more than conventional cost minimization. Nevertheless, they primarily develop optimal or near-optimal policies for stochastic inventory systems, whereas comparative applications of established lot-sizing methods often remain focused on inventory-cost outcomes. Relatively limited attention has been given to translating the results of classical lot-sizing methods into a structured policy framework that simultaneously considers cost performance and compliance with a monthly purchasing ceiling.

Against this background, this study develops a liquidity-aware framework for evaluating and selecting among established lot-sizing methods. The framework extends conventional comparisons beyond TIC by examining the monthly purchasing-expenditure profiles generated by Wagner-Whitin (WW), Lot-for-Lot (L4L), Economic Order Quantity (EOQ), Periodic Order Quantity (POQ), and Part-Period Balancing (PPB). Specifically, the study: (i) compares the TIC generated by the five methods across 13 engine-oil SKUs; (ii) statistically tests the differences among them; (iii) evaluates liquidity exposure through violation frequency and cumulative excess under monthly purchasing ceilings of IDR 40 million, IDR 50 million, and

IDR 60 million; and (iv) translates the resulting cost and liquidity profiles into cost-first, cash-first, and balanced policy alternatives while examining whether the recommended methods change under different levels of financial flexibility. The contribution therefore lies not in proposing a new lot-sizing algorithm, but in broadening the basis on which established methods are evaluated and selected through a transparent two-stage framework that integrates inventory-cost performance with purchasing-liquidity exposure.

Literature Review

Inventory Management

Inventory management is essential for maintaining service reliability while coping with financial limitations, a concern that is particularly critical for small and medium-sized enterprises (SMEs). According Silver et al. (2016), it involves planning and monitoring stock so that products remain available at the lowest feasible cost, which requires balancing holding expenses, ordering costs, and service performance. Mor et al. (2021) show that weak inventory control often produces either surplus items that turn into dead stock or shortages that disappoint customers and erode profitability. Wang et al. (2024) argue that an optimal inventory management strategy involves balancing ordering and holding costs while accounting for supply chain uncertainty and demand fluctuations. In practice, Drakeley & Perera (2022) emphasize that behavioral biases in ordering decisions further underscore the need for disciplined, transparent lot-sizing and replenishment rules in inventory management.

Lot-Sizing Methods in Inventory Control

The technical definitions and calculation procedures of WW, L4L, EOQ, POQ, and PPB follow established inventory-management references, particularly Heizer et al. (2017). These sources are used to establish the methodological foundations of the five methods, while their comparative performance and practical applicability are evaluated using more recent empirical studies from different operational settings.

Wagner-Whitin Method

The Wagner-Whitin (WW) method applies dynamic programming to deterministic but time-varying demand. It evaluates alternative replenishment timings across the planning horizon and selects the combination that minimizes total ordering and holding costs. Its principal advantage is the ability to consider inter-period cost consequences rather than determining orders one period at a time. Recent developments show its growing use in practice, particularly through spreadsheet-based applications that integrate visualization and sensitivity analysis to enhance accuracy and speed in decision-making. Hakim & Ardiansyah (2025) designed a spreadsheet tool for WW that outperformed manual approaches by reducing errors and accelerating computation. In the spare parts sector, Adipradana & Muharni (2021) applied WW after ABC classification at PT XYZ and documented annual cost savings of 15.29% and 21.66% for two materials, while also achieving more stable order frequencies. A comparative study by Asmal et al. (2023) in the garment industry contrasted WW with the Silver-Meal heuristic, showing that Silver-Meal generated lower costs in most cases, although WW was optimal for one item with a particular demand profile. These studies confirm that WW can be highly effective, but its relative advantage depends on the specific demand patterns and cost structures involved.

Lot-for-Lot Method

The Lot-for-Lot (L4L) method sets the order quantity equal to the net requirement of each period. This rule minimizes planned inventory carried between periods, but it may generate a high ordering frequency when demand occurs in many successive periods. Its cost performance therefore depends largely on whether the savings in holding costs are sufficient to offset repeated ordering costs. Ventura & Lu (2023) tested L4L against an order frequency policy in a two-stage supply chain model and found that both approaches reduced overall costs, although L4L was less favorable when setup costs were high relative to holding costs. In shipbuilding, Baroroh et al. (2024) demonstrated that L4L reduced TIC to 28%, better than EOQ (29%) and the company's current method (43%), showing strong efficiency for expensive materials. Meanwhile, Florim et al. (2019) compared ten lot-sizing methods and concluded that while L4L is simple and effective for minimizing stock, its performance remains highly dependent on the demand environment. Hence, L4L often serves as a baseline method but is not always the most cost-effective solution.

Economic Order Quantity Method

The Economic Order Quantity (EOQ) method determines a fixed order quantity by balancing ordering and holding costs. Its conventional logic is most applicable when demand and cost conditions are relatively stable, although modified and probabilistic versions have also been employed in more variable operating environments. Empirical studies continue to show the practical relevance of EOQ. Under a lumpy demand setting, Nurprihatin et al. (2022) integrate ARIMA based forecasting into an MRP framework and compare probabilistic EOQ with Periodic Order Quantity (POQ) across selected raw materials after ABC classification. The study concludes that POQ yields the lowest total cost for S45C-F, SGT-R, and SKD11-R, whereas SLD-R is best planned with probabilistic EOQ, underscoring that the lot-sizing choice is item specific rather than universally dominant. Khamai & Sopadang (2023) applied EOQ at an automotive parts manufacturer after screening items for demand variability (ABC). Items with stable demand were managed using EOQ, whereas unstable items were handled with heuristics such as Silver-Meal and PPB. Compared with the firm's prior practice, total raw material inventory costs fell by 44.58% (2019), 44.52% (2020), and 45.55% (2021), underscoring EOQ's effectiveness when demand is predictable. In the healthcare sector, Saddikuti et al. (2024) reported that EOQ, when paired with a (Q, r) policy, provided a reliable framework for managing surgical supplies by reducing shortages while avoiding excess inventory.

Periodic Order Quantity Method

Periodic Order Quantity (POQ) converts EOQ logic into a replenishment interval. Instead of ordering a constant quantity, the method determines the approximate number of periods covered by each order and adjusts the order quantity according to the requirements within that interval. This structure can provide a more regular purchasing schedule, but it may also concentrate several periods of demand into a single replenishment. Studies after 2018 show that POQ's performance is highly context specific. In a motorcycle chain manufacturer, Kurniawan & Raphaeli (2018) reported that POQ occupied a middle ground, with costs lower than EOQ but higher than L4L. Arif & Sukarno (2021) tested EOQ, POQ, and Min-Max in a cement bag case and found EOQ to be the least costly, with POQ viable but more expensive. Saptadi et al. (2023) examined raw materials in cement production and concluded that L4L yielded the lowest inventory costs under both theoretical and actual purchasing conditions, while POQ served as a comparator for order frequency. Together, these findings suggest that POQ fits situations where managers prefer standardized replenishment cycles, though its cost-effectiveness is not guaranteed across contexts.

Part-Period Balancing Method

Part-Period Balancing (PPB) is a heuristic that extends the coverage of an order until the accumulated holding cost approaches the cost of placing an additional order. The method therefore seeks a practical balance between frequent replenishment and excessive inventory accumulation. Compared with WW, PPB uses a simpler period-by-period decision rule and does not evaluate every possible replenishment combination across the full planning horizon. Recent studies confirm PPB's competitive performance. Rafsanjana et al. (2024) showed that PPB generated the lowest inventory cost for MSG in a food based MSME. In a coffee industry case, Maharani et al. (2025) reported that PPB and WW were among the most efficient in reducing ordering costs while maintaining availability. A broader comparative analysis by Florim et al. (2019) further demonstrated that PPB performs near optimally under fluctuating or seasonal demand, supporting its role as a robust heuristic across manufacturing contexts. Overall, PPB is particularly relevant in environments with unstable demand, as it offers a practical balance between cost efficiency and operational feasibility. Nevertheless, its effectiveness still depends on demand variability and cost structure, meaning it does not always guarantee the lowest possible inventory cost.

Synthesis of Prior Lot-Sizing Evidence

Taken together, previous studies do not identify a universally dominant lot-sizing method. WW tends to perform strongly when deterministic demand varies across periods and when evaluating cost interactions across the full planning horizon provides meaningful savings. PPB often approximates a similar ordering-holding cost trade-off through a simpler heuristic procedure. L4L limits inventory carried between periods but can become costly when repeated replenishment incurs substantial fixed ordering expenses. EOQ provides a simple fixed-quantity rule that is most suitable under relatively stable demand, while POQ offers regular ordering intervals but may consolidate expenditures into selected periods.

The variation in findings across empirical studies indicates that lot-sizing performance depends on the interaction among demand patterns, ordering costs, holding costs, and operational preferences. Most prior comparisons, however, evaluate the methods primarily through inventory cost, order frequency, or inventory availability. Less attention has been given to whether the purchasing schedules generated by these methods remain manageable under recurring working-capital limitations. This evidence indicates a remaining need to examine whether cost-efficient lot-sizing schedules also produce manageable purchasing-expenditure profiles under recurring working-capital limitations.

Total Inventory Cost (TIC)

Anil Kumar & Suresh (2008) explain that TIC provides a comprehensive measure of inventory efficiency, since it captures the trade-off between ordering and holding while ensuring that customer service standards are upheld. Empirical studies also confirm the centrality of TIC as a performance indicator. Arif & Sukarno (2021) compared company practice with EOQ, POQ, and Min-Max methods, showing that EOQ produced the lowest TIC and was therefore recommended as the more efficient approach. In another case, Fithri et al. (2019) applied EOQ to gypsum inventory at PT Semen Padang and demonstrated that TIC captures not only the optimal order quantity but also order frequency and total annual cost. Collectively, these findings reinforce both the theoretical and empirical basis that the method delivering the lowest TIC while still ensuring timely availability and sustaining cashflow represents the most relevant candidate for evaluation in this study.

Repeated Measures Analysis

Ho (2006) describes a repeated measures design as one in which the same subjects are observed under multiple conditions, allowing the researcher to detect differences while holding subject specific factors constant. Because between subject variation is eliminated, the test becomes more sensitive to treatment effects. In this study, the design is fully within-subjects: the 13 oil items are each evaluated under five inventory control methods (WW, L4L, EOQ, POQ, and PPB).

According Pituch & Stevens (2016), repeated measures analysis is valid if three main assumptions are met: (1) multivariate normality in each condition, (2) sphericity, or equality of the variance-covariance of difference scores across conditions, which can be tested using Mauchly's test, and (3) independence between subjects. If the assumption of sphericity is violated, Greenhouse-Geisser or Huynh-Feldt corrections are applied to adjust the degrees of freedom and maintain the validity of the F-test. The question of normality has also received attention in the literature. Blanca et al. (2023) demonstrated through Monte Carlo simulations that repeated measures analysis is relatively robust to violations of normality, provided that the assumption of sphericity is satisfied. Their results indicate that Type I error rates and the power of the F-test generally remain stable even when the data distribution departs from normality. Therefore, although it remains important to check for normality, its violation does not necessarily invalidate the results of repeated measures analysis.

Inventory Planning and Financial Feasibility

Traditional inventory and lot-sizing methods generally determine replenishment quantities by considering demand requirements, ordering costs, and holding costs. This approach commonly assumes that a purchasing schedule that is operationally and economically justified can also be financed when the order is placed. For small and medium-sized enterprises (SMEs), however, the availability and timing of internal funds may limit the implementation of such schedules, particularly when a method consolidates purchasing requirements into several high-expenditure periods. Consequently, a replenishment plan that minimizes inventory-related costs is not necessarily the plan that creates the most manageable working-capital profile.

Previous studies have established the importance of connecting inventory decisions with financial conditions. Katehakis et al. (2016) develop a dynamic model in which inventory, cash, borrowing, and deposits are managed jointly under stochastic demand. Their findings show that the optimal replenishment decision depends on the firm's net worth and the relative financial consequences of borrowing or retaining cash. Chen and Rossi (2021) examine a single-item, multi-period stochastic lot-sizing problem in which available cash creates an endogenous upper bound on the quantity that can be ordered. Their heuristic (s,C(x),S) policy demonstrates that an ordering decision depends jointly on inventory availability, cash availability, and fixed ordering costs. Extending the analysis to a multi-product setting, Chen and Zhang (2024) show that delayed revenue collection, limited initial cash, and access to order-based financing influence purchasing decisions when several products draw from a common cash pool.

Collectively, these studies demonstrate that replenishment decisions cannot always be evaluated independently from financial conditions. Nevertheless, their primary focus is on embedding cash availability or financing mechanisms directly into dynamic stochastic optimization models in order to determine optimal or near-optimal ordering and financing decisions. However, these studies do not directly examine how multiple established deterministic lot-sizing methods perform when their resulting purchasing schedules are evaluated against a common recurring managerial purchasing ceiling.

Comparative studies of classical lot-sizing methods, by contrast, generally emphasize Total Inventory Cost and the trade-off between ordering and holding costs. This leaves a more specific practical gap concerning how SMEs can evaluate competing lot-sizing alternatives when a cost-efficient method produces concentrated monthly purchasing expenditures. In such circumstances, managers require information not only about the cost generated by each method, but also about the frequency and magnitude of its exposure to periodic liquidity limits.

The present study addresses this gap by separating lot-sizing calculation from financial-feasibility evaluation. WW, L4L, EOQ, POQ, and PPB are first applied using their established procedures to generate purchasing schedules and TIC outcomes. The resulting periodic purchasing expenditures are subsequently evaluated using violation frequency and cumulative excess relative to a recurring managerial purchasing ceiling. Thus, the financial ceiling is not imposed as a hard constraint within the original lot-sizing algorithms, but is used as a benchmark in the subsequent policy-selection process. The resulting cost and liquidity profiles are translated into cost-first, cash-first, and balanced policy alternatives. Accordingly, the study contributes a liquidity-aware evaluation framework for selecting among established lot-sizing methods rather than proposing a new cash-constrained ordering algorithm.

Policy Selection through Binary Linear Programming

Policy selection frequently requires decision-makers to choose among several alternatives that exhibit conflicting cost and feasibility characteristics. Binary linear programming provides a suitable structure for this type of decision because each alternative can be represented by a binary variable that takes the value of one when the alternative is selected and zero otherwise. A selection constraint can then ensure that only one alternative is chosen, while the objective function and additional constraints represent the managerial priority under consideration. Williams (2013) explains that binary decision variables are particularly useful for modelling mutually exclusive choices and logical conditions within optimization problems.

The application of optimization models to decisions involving multiple operational criteria has been demonstrated in several empirical contexts. Riskadayanti et al. (2020) developed a Mixed-Integer Linear Programming model for production planning in the sawn-timber industry, demonstrating how production decisions can be optimized while accounting for cost and capacity limitations. Garside and Kristiandy (2013) combined the Analytic Hierarchy Process with Preemptive Goal Programming to select suppliers according to prioritized criteria, including cost, quality, and delivery performance. Their study illustrates how managerial priorities can be represented hierarchically when all objectives cannot be optimized simultaneously. Similarly, Anis et al. (2007) applied Goal Programming to production planning involving cost, revenue, and machine-utilization objectives, showing the usefulness of optimization techniques for reconciling competing goals under limited resources. Although these studies address different operational settings, they provide methodological support for using optimization-based selection when decision alternatives must be evaluated against several potentially conflicting criteria.

In lot-sizing decisions, a method that produces the lowest Total Inventory Cost may not necessarily generate the most manageable pattern of monthly purchasing expenditures. Conversely, a method that performs well in terms of liquidity exposure may produce a higher inventory cost. This creates a policy-selection problem rather than a single-objective cost-minimization problem. The present study therefore uses binary decision variables to translate the cost and liquidity profiles of the five lot-sizing methods into three managerial orientations. The cost-first orientation prioritizes the method with the lowest TIC. The cash-first orientation prioritizes compliance with the monthly purchasing ceiling by emphasizing the lowest liquidity exposure. The balanced orientation considers cost efficiency while limiting the frequency and magnitude of purchasing-ceiling exceedances.

The monthly ceiling is not imposed directly within the original calculations of WW, L4L, EOQ, POQ, and PPB. Instead, it serves as a financial-feasibility benchmark for evaluating the purchasing schedules generated by these methods and for formulating the subsequent policy-selection model. Accordingly, binary linear programming is used as a structured mechanism for selecting among established lot-sizing alternatives under different managerial priorities, rather than for modifying the internal algorithms of the five methods.

Research Methodology

Research Design

This study adopts a quantitative comparative case-study design that integrates lot-sizing simulation, statistical testing, and structured policy selection. Five established lot-sizing methods namely Wagner-Whitin (WW), Lot-for-Lot (L4L), Economic Order Quantity (EOQ), Periodic Order Quantity (POQ), and Part-Period Balancing (PPB) were applied to the same monthly demand data for 13 engine-oil SKUs. Using identical demand and cost parameters for all methods, POM-QM for Windows generated the replenishment schedules and Total Inventory Cost (TIC) outcomes. This common-data structure allows the performance differences to be attributed to the replenishment logic of the respective methods.

The analysis was conducted in three stages. In the first stage, the five methods were compared according to TIC, defined as the sum of ordering and holding costs. In the second stage, repeated-measures analysis was used to determine whether the TIC differences were statistically significant. Because each of the 13 SKUs was evaluated under all five methods, the methods represent within-subject conditions and the SKUs serve as the repeated observational units. This design controls for item-specific differences and provides stronger inferential evidence than a descriptive cost ranking alone.

In the third stage, the purchasing schedules generated by the five methods were evaluated from a liquidity perspective. A monthly purchasing ceiling of IDR 50 million was used as the base-case managerial benchmark. To assess the sensitivity of the results to different working-capital conditions, alternative ceilings of IDR 40 million and IDR 60 million were also examined. For each scenario, liquidity exposure was measured through violation frequency, defined as the number of months in which purchasing expenditure exceeded the ceiling, and cumulative excess, defined as the total amount above the ceiling across the planning horizon.

The purchasing ceilings were not incorporated as constraints within the original WW, L4L, EOQ, POQ, or PPB calculations. Instead, they were applied after the replenishment schedules had been generated, thereby preserving the original procedures of the five methods. A binary linear programming model was then used to select exactly one method under three managerial orientations. The cost-first policy minimizes TIC, the cash-first policy minimizes cumulative excess, and the balanced policy selects the lowest-TIC method among alternatives satisfying predefined limits for TIC, violation frequency, and cumulative excess.

Accordingly, the research design is both comparative and decision-oriented. It evaluates whether the established lot-sizing methods differ in inventory-cost performance and subsequently translates their cost and liquidity profiles into policy recommendations under alternative levels of financial flexibility. This sequential separation between lot-sizing calculation, statistical evaluation, and liquidity-based policy selection provides a transparent framework that can be adapted to different working-capital priorities.

Object of Study

The empirical data were obtained from IM, a motorcycle workshop in Indonesia. The population consists of 65 engine oil SKUs recorded in the workshop's transaction system from January to December 2024. Applying the Pareto principle, 13 SKUs were selected as the study sample since together they represented about 78.53% of cumulative demand. Each of these SKUs contributed more than 2% of total oil demand in 2024, as detailed in Table 1.

The ordering cost was estimated at IDR 550,000 per replenishment, while the holding cost was estimated at IDR 1,097 per unit.

Table 1.
Study Sample (2024)

Engine-Oil	$\sum Demand$	Turnover (%)
Oli Gear Box 120ml	2.293	11,33%
Oli MPX2 0,8l	2.897	14,31%
Oli Gear Box 100ml	1.110	5,48%
Oli Yamalube Matic 0,8l	3.239	16,00%
Oli MPX2 0,65l	433	2,14%
Oli Yamalube Sport 1l	862	4,26%
Oli Ultratec 0,8l	1.249	6,17%
Oli Yamalube Silver 0,8l	1.160	5,73%
Oli Ultratec Matic 0,8l	526	2,60%
Oli Castrol Activ Matic 0,8l	763	3,77%
Oli Gear Box 140ml	427	2,11%
Oli MPX1 0,8l	432	2,13%
Oli Castrol Activ Essential 0,8l	500	2,47%
Total	15.904	78,53%

Sumber: IM Workshop

The cost parameters were derived from the workshop's procurement and financial records and were applied consistently across all five lot-sizing methods.

Historical purchasing expenditure for the 13 selected SKUs averaged approximately IDR 44.3 million per month and ranged from IDR 36.1 million to IDR 51.8 million. An IDR 50 million monthly purchasing ceiling was therefore used as the base-case managerial benchmark. Alternative ceilings of IDR 40 million and IDR 60 million were examined to represent tighter and more flexible working-capital conditions.

The case is used as an empirical setting for examining how established lot-sizing methods produce different inventory-cost and purchasing-expenditure profiles. The numerical recommendations remain specific to the demand, cost, and financial conditions of IM Workshop, while the evaluation procedure may be adapted by SMEs facing similar inventory and working-capital trade-offs.

Lot-sizing Models

The lot-sizing formulations used in this study follow established references in the inventory management literature, particularly Heizer et al. (2017), Silver et al. (2016), and Florim et al. (2019), and are summarized in Table 2. Rather than deriving closed-form solutions, the analysis relies on computational routines within POM-QM to generate outcomes. In this setup, ordering costs are triggered whenever a purchase is made, while holding costs accumulate from units carried forward into subsequent periods.

Let D denote total demand, either expressed on an annual basis or disaggregated by period as Dt. The average demand per period is represented by D Order quantities are denoted by Qt for a given period, with Q^* referring to the optimal order size obtained from the **EOQ** formulation. Ordering or setup cost is indicated by S (or equivalently K), while the unit holding cost is represented by H when expressed annually, or h when expressed on a per-period basis.

In the case of the POQ method, *T* represents the number of periods of demand covered by each replenishment. For the Wagner-Whitin algorithm, *F(t)* denotes the minimum cumulative cost up to period *t*, where decision indices *j, t, u* identify the timing of replenishment and associated demand coverage.

In this study, Total Inventory Cost (TIC) was operationalized as the sum of ordering and holding costs generated by each replenishment schedule. Purchase expenditure was excluded from TIC because it represents the monetary value of the products purchased rather than the ordering-holding cost consequences of the lot-sizing decision. Monthly purchasing expenditure was therefore evaluated separately in the subsequent liquidity analysis.

Table 2.
Summary of Lot-Sizing Methods

Acronym	Method Name	Formula(s)	Main Comments
WW	Wagner–Whitin	$F(t) = \min_{1 \le j \le t}\{F(j-1) + K + h\sum_{u=j}^{t}(u-j)D_u\}, F(0) = 0$	Dynamic programming; optimal benchmark for deterministic variable demand
L4L	Lot-for-Lot	$Q_t = D_t$	Orders exactly match demand each period; no holding cost, but high order frequency
EOQ	Economic Order Quantity	$Q^* = \sqrt{\frac{2DS}{H}}$	Minimizes annual total inventory cost under constant demand
POQ	Periodic Order Quantity	$T = \frac{Q^*}{\bar{D}}$ (number of periods per order)	Adapts EOQ to discrete periods; aligns replenishment with planning cycles
PPB	Part-Period Balancing	Extend order horizon until: $Holding\ cost \ge Ordering\ Cost$	Heuristic; balances setup vs. holding costs period-by-period; near-optimal outcome

Sumber: Adapted by the authors from Heizer et al. (2017), Silver et al. (2016), and Florim et al. (2019)

Analytical Tools
Lot-sizing calculations were first conducted using POM-QM for Windows, which provides automated modules for the WW, L4L, EOQ, POQ, and PPB methods. This ensured consistent cost estimation and reproducibility across all models.

The second stage involved statistical testing in SPSS. Following Ho (2006) and Pituch & Stevens (2016), repeated measures analysis was used to assess whether differences in Total Inventory Cost (TIC) among the five methods were statistically significant.

Post-hoc pairwise comparisons were carried out with Bonferroni adjustment as suggested by Ho (2006), and Greenhouse-Geisser corrections were applied whenever the sphericity assumption was not satisfied.

Finally, linear programming formulations for policy evaluation were implemented in Microsoft Excel using the Solver add-in. Binary decision variables were employed to model the selection of exactly one lot-sizing method, consistent with the integer programming framework described by Williams (2013).

This allowed the integration of TIC outcomes with financial feasibility indicators (excess and violations), thereby operationalizing the cost-first, cash-first, and balanced policies defined in this study.

Research Procedure

The research procedure consisted of five sequential stages, as illustrated in Figure 2. First, transaction and inventory data for 65 engine-oil SKUs recorded from January to December 2024 were compiled, and Pareto analysis was used to select 13 high-turnover SKUs representing approximately 78.53% of total engine-oil demand.

Second, WW, L4L, EOQ, POQ, and PPB were applied to the selected SKUs using identical demand, ordering-cost, and holding-cost parameters. This stage generated the replenishment schedules and Total Inventory Cost (TIC) outcomes for each method.

Third, repeated-measures analysis was conducted to determine whether the TIC differences among the five methods were statistically significant. Appropriate corrections and pairwise comparisons were applied based on the statistical assumptions described in the Analytical Tools subsection.

Fourth, the monthly purchasing expenditures generated by each method were evaluated under purchasing-ceiling scenarios of IDR 40 million, IDR 50 million, and IDR 60 million. Violation frequency and cumulative excess were calculated for each method and scenario, after which the binary selection model was applied under the cost-first, cash-first, and balanced orientations. The purchasing ceilings did not alter the original replenishment schedules or TIC outcomes.

Finally, the policy-selection outcomes were compared across the three scenarios to assess their stability under different levels of financial flexibility. The results were then translated into managerial recommendations concerning the trade-off between inventory-cost efficiency and purchasing-liquidity exposure.

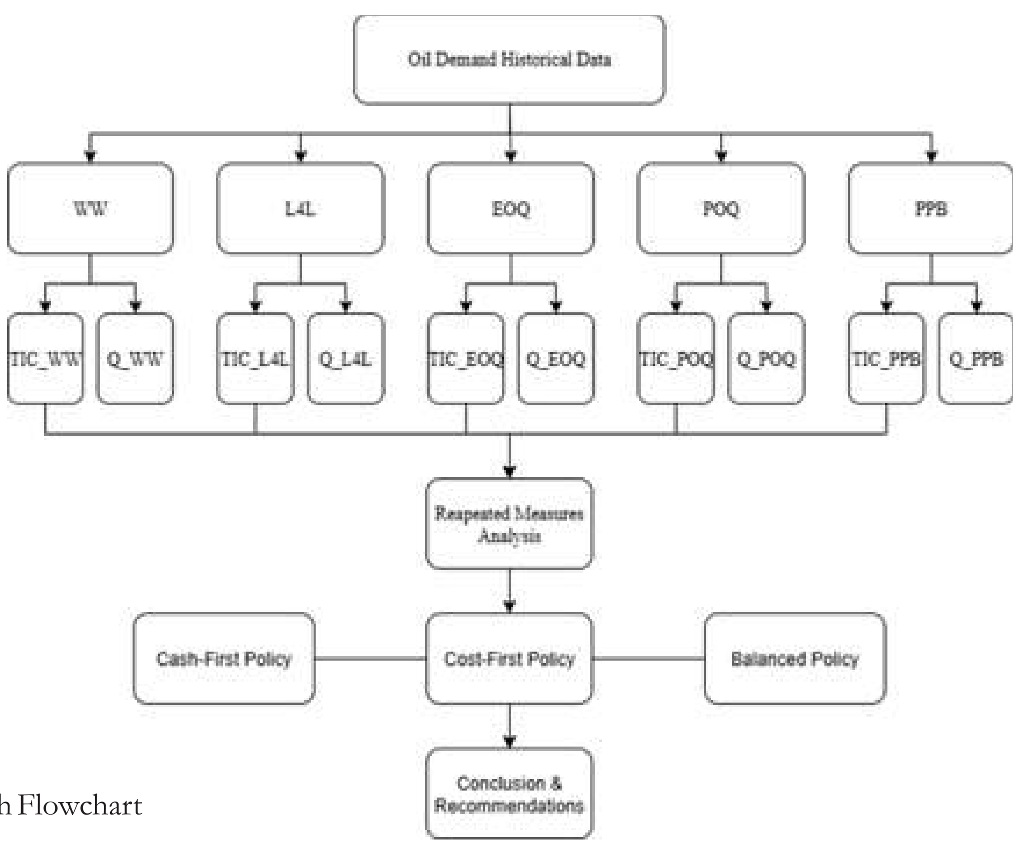

Figure 2.
Research Flowchart

Policy Formulation under Cash Constraint

After the replenishment schedules had been generated, a binary linear programming model was used to select exactly one lot-sizing method under each managerial orientation. As noted by Williams (2013), linear programming with binary decision variables was used to ensure that exactly one lot-sizing method was selected. Each lot-sizing method *m* (where *m* = 1, 2, … , M) was evaluated by three indicators: the total inventory cost *(Cm)*, the total cash excess *(Em)*, and the number of cash violations *(Vm)*. A binary variable z_m is assigned to each method, where z_m=1 indicates that method *m* is selected, and z_m=0 indicates rejection. The condition $\sum_m^M z_m = 1$ guarantees that exactly one method is chosen. Based on this indicators, three decision policies were defined:

(1) Cost-first policy

This policy represents the traditional rule of minimizing the total inventory cost (TIC), regardless of whether or not the monthly ceiling is breached. Formally:

$$min\ Z = \sum_{m=1}^{M} C_m z_m$$

Subject to:

$$\sum_{m=1}^{M} z_m = 1, z_m \in \{0,1\}(m = 1, \ldots, M)$$

((2) Cash-first policy

The decision rule is to prioritize financial compliance by directly selecting the method that produces the lowest cash excess. The policy ensures that the option chosen is the one that best fits the cash ceiling, even if its total cost is not the lowest:

$$min\ Z = \sum_{m=1}^{M} E_m z_m$$

Subject to:

$$\sum_{m=1}^{M} z_m = 1, z_m \in \{0, 1\}$$

(3) Balanced policy

This policy introduces a more restrictive approach by requiring that all three indicators include cost, excess, and violations remain within predetermined limits. Only methods that satisfy these constraints are eligible, and among them, the one with the lowest total cost is selected. This reflects the treatment of multiple hard constraints in mathematical programming, as discussed by Williams (2013):

$$min\ Z = \sum_{m=1}^{M} C_m z_m$$

Subject to:

$$\sum_{m=1}^{M} z_m = 1,$$

$$\sum_{m=1}^{M} E_m z_m \le \bar{E}, \sum_{m=1}^{M} V_m z_m \le \bar{V}, \sum_{m=1}^{M} C_m z_m \le \bar{C}, \qquad z_m \in \{0, 1\}$$

The tolerance parameters used in the balanced policy were established as data-informed analytical benchmarks rather than universal values prescribed by the literature or formal company rules. The maximum TIC was set at IDR 55 million based on the mean TIC across the five methods, which was approximately IDR 52.64 million, and was rounded upward to provide a practical cost allowance. The maximum violation frequency was set at five periods, corresponding to the median number of violations across the evaluated methods and ensuring that the purchasing ceiling was satisfied in the majority of the 12 monthly periods.

The maximum cumulative excess was set at IDR 130 million. The mean cumulative excess across the five methods under the IDR 50 million base-case ceiling was approximately IDR 117.22 million. A 10% analytical tolerance produced a value of approximately IDR 128.95 million, which was rounded to IDR 130 million. This allowance was intended to represent moderate liquidity exposure while excluding methods with substantially concentrated purchasing requirements.

Accordingly, the balanced-policy thresholds are case-specific screening parameters used to illustrate a compromise between inventory-cost efficiency and liquidity exposure. They are not proposed as universally optimal limits. In practical applications, managers may revise these parameters according to available working capital, risk tolerance, supplier-credit arrangements, and other financial conditions.

Sensitivity Analysis

A one-way sensitivity analysis was conducted to examine whether the policy recommendations remained consistent under alternative liquidity conditions. The monthly purchasing ceiling was varied across three scenarios: IDR 40 million for the low-liquidity scenario, IDR 50 million for the base-case scenario, and IDR 60 million for the high-liquidity scenario. The alternative values represent a 20% decrease and increase from the base-case ceiling.

For each ceiling scenario, violation frequency and cumulative excess were recalculated from the monthly purchasing schedules generated by WW, L4L, EOQ, POQ, and PPB. The three policy-selection rules were then reapplied. The cost-first policy selected the method with the lowest TIC, the cash-first policy selected the method with the lowest cumulative excess, and the balanced policy selected the lowest-cost method among alternatives satisfying the predefined limits for TIC, violation frequency, and cumulative excess. This analysis was intended to assess the robustness of the policy orientations and determine whether changes in liquidity conditions altered the recommended lot-sizing method.

The balanced-policy thresholds were held constant across all scenarios. The original replenishment schedules and TIC outcomes also remained unchanged because the purchasing ceilings served only as subsequent evaluation benchmarks.

Results and Discussion

Simulation Results

The five lot-sizing methods generated different replenishment schedules and Total Inventory Cost (TIC) outcomes across the 13 engine-oil SKUs. As summarized in Table 3, WW produced the lowest aggregate TIC of IDR 40.22 million, followed by PPB at IDR 42.42 million, POQ at IDR 48.56 million, EOQ at IDR 53.85 million, and L4L at IDR 78.16 million. The detailed SKU level calculations and monthly replenishment schedules were generated using POM-QM under identical demand, ordering cost, and holding cost parameters.

Statistical Analysis

To determine whether the observed cost differences were statistically meaningful, a repeated measures analysis of TIC was conducted. Mauchly's test of sphericity showed that the assumption of sphericity was violated, $\chi^2(9) = 58.849$, $p < .001$ as shown in figure 3.

Consequently, the within subject effects was interpreted using the Greenhouse-Geisser correction ($\varepsilon = .36$), following the procedure recommended by Ho (2006) and Pituch & Stevens (2016). The corrected omnibus test revealed that the choice of lot-sizing method had a statistically significant effect on TIC, $F(1.438, 17.258) = 45.853$, $p < .001$ as shown in figure 4. This confirms that, across the 13 SKUs, the mean costs varied systematically among the five methods. Pairwise comparisons with Bonferroni adjustment further revealed that most of these differences were significant.

The pairwise comparisons as shown in figure 5 showed that WW generated significantly lower TIC than L4L, EOQ, and PPB, but did not differ significantly from POQ. PPB generated significantly lower TIC than L4L and EOQ, was significantly more costly than WW, and did not differ significantly from POQ. POQ generated significantly lower TIC than L4L and EOQ, but did not differ significantly from

WW or PPB. EOQ was significantly less costly than L4L, significantly more costly than WW and PPB, and did not differ significantly from POQ. L4L generated significantly higher TIC than all other methods. The ranking of methods based on TIC outcomes is presented in Table 4.

These results indicate that the descriptive TIC ranking should be interpreted together with the pairwise comparisons. Although PPB produced the second-lowest aggregate TIC, its TIC remained statistically higher than that of WW. POQ ranked third descriptively but was not statistically different from either WW or PPB.

Table 3.
Simulation results of order quantities and Total Inventory Cost (TIC) per SKU across five lot-sizing methods

Brand	Method	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total	TIC
Oli Gear Box 120ml	WW	238		377		443		443		405			286	2.192	4.431.007
	L4L	62	176	206	171	214	229	236	207	193	176	36	286	2.192	6.600.000
	EOQ	438		438			438		438		438		438	2.628	6.526.277
	POQ	339		377		443		443		369			286	2.257	5.602.603
	PPB	444			614			636			498			2.192	4.899.717
Oli MPX2 0,8l	WW	323		550		448		523		477		457		2.778	4.712.936
	L4L	114	209	361	189	303	145	238	285	279	198	195	262	2.778	6.600.000
	EOQ	492		492		492		492		492		492		2.952	6.626.104
	POQ	442		550		448		523		477		457		2.897	6.279.452
	PPB	323		853			383		762			457		2.778	5.140.363
Oli Gear Box 100ml	WW			332					348			293		973	2.880.834
	L4L			12	130	126	64		102	135	111	115	178	973	4.964.261
	EOQ			305			305				305		305	1.220	4.263.457
	POQ			272			166			361			178	977	3.275.060
	PPB			332					348			293		973	2.880.834
Oli Yamalube Matic 0,8l	WW	413		545		560		659		489		390		3.056	4.946.597
	L4L	221	192	246	299	290	270	303	356	281	208	214	176	3.056	6.600.000
	EOQ	520		520		520		520	520				520	3.120	6.377.085
	POQ	596		545		560		659		489		390		3.239	7.355.609
	PPB	659			589		573		845			390		3.056	5.108.550

Table 3. (Continued)

Brand	Method	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total	TIC
Oli MPX2 0,65l	WW					242						191		433	1.619.978
	L4L					54	75	73	40			58	133	433	3.300.000
	EOQ					190		190					190	570	2.753.582
	POQ					242						191		433	1.619.978
	PPB					242						191		433	1.619.978
Oli Yamalube Sport 1l	WW	246					321				276			843	3.133.144
	L4L	22	53	68	44	59	102	67	71	81	62	85	129	843	6.600.000
	EOQ	268					268			268			268	1.072	4.127.429
	POQ	206				299				357				862	3.423.849
	PPB	246					321				276			843	3.133.144
Oli Ultratec 0,8l	WW	287				311			259		339			1.196	3.536.146
	L4L	11	102	136	38	105	113	93	147	112	132	95	112	1.196	6.600.000
	EOQ	323				323			323		323			1.292	4.413.746
	POQ	302				311			391			207		1.211	3.796.135
	PPB	287				311			391			207		1.196	3.598.675
Oli Yamalube Silver 0,8l	WW	117		401				350				179		1.047	3.494.460
	L4L	9	108	140	99	73	89	90	121	90	49	67	112	1.047	6.600.000
	EOQ	311			311				311			311		1.244	4.411.552
	POQ	370				252			260			179		1.061	3.759.934
	PPB	257			351				327				112	1.047	3.623.906
Oli Ultratec Matic 0,8l	WW			262						197				459	2.291.342
	L4L			19	45	43	84	42	29	48	33	32	84	459	5.544.977
	EOQ			210				210					210	630	3.019.056
	POQ			235					226					461	2.370.326
	PPB			233					226					459	2.348.386

Table 3. (Continued)

Brand	Method	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total	TIC
Oli Castrol Activ Matic 0,8l	WW	243						225			208			676	2.763.455
	L4L	3	59	54	109	10	8	77	69	79	75	52	81	676	6.600.000
	EOQ	253						253			253			759	3.227.486
	POQ	312						300					81	693	3.569.750
	PPB	225				164				287				676	3.059.645
Oli Gear Box 140ml	WW				137			286						423	1.838.281
	L4L				23	40	74	115	49	26	34	28	34	423	4.950.000
	EOQ				189			189				189		567	2.677.889
	POQ				301					122				423	2.110.337
	PPB				252				171					423	2.029.159
Oli MPX1 0,8l	WW	173						205						378	2.215.649
	L4L	15	43	20	30	40	25	34	25	20	42	52	32	378	6.600.000
	EOQ	190						190						380	2.340.707
	POQ	202						173					32	407	2.971.885
	PPB	173						205						378	2.215.649
Oli Castrol Activ Essential 0,8l	WW	223						207						430	2.358.259
	L4L	3	42	45	62	38	33	35	29	43	32	39	29	430	6.600.000
	EOQ	204					204						204	612	3.085.973
	POQ	260							172					432	2.426.273
	PPB	190					211						29	430	2.763.455

Mauchly's Test of Sphericity[a]

Measure: MEASURE_1

Within Subjects Effect	Mauchly's W	Approx. Chi-Square	df	Sig.	Epsilon[b] Greenhouse-Geisser	Huynh-Feldt	Lower-bound
Metode	.004	58.849	9	.000	.360	.395	.250

Tests the null hypothesis that the error covariance matrix of the orthonormalized transformed dependent variables is proportional to an identity matrix.

a. Design: Intercept
Within Subjects Design: Metode

b. May be used to adjust the degrees of freedom for the averaged tests of significance. Corrected tests are displayed in the Tests of Within-Subjects Effects table.

Figure 3.
SPSS Output of Mauchly's Test of Spheric

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.
Metode	Sphericity Assumed	7.138E+13	4	1.785E+13	45.853	.000
	Greenhouse-Geisser	7.138E+13	1.438	4.963E+13	45.853	.000
	Huynh-Feldt	7.138E+13	1.581	4.515E+13	45.853	.000
	Lower-bound	7.138E+13	1.000	7.138E+13	45.853	.000
Error(Metode)	Sphericity Assumed	1.868E+13	48	3.892E+11		
	Greenhouse-Geisser	1.868E+13	17.258	1.082E+12		
	Huynh-Feldt	1.868E+13	18.970	9.848E+11		
	Lower-bound	1.868E+13	12.000	1.557E+12		

Figure 4.
SPSS Output for Test of Within-Subjects

Pairwise Comparisons

Measure: MEASURE_1

(I) Metode	(J) Metode	Mean Difference (I-J)	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b] Lower Bound	Upper Bound
1	2	-2918242.31*	261972.459	.000	-3816400.277	-2020084.339
	3	-1048327.31*	151940.018	.000	-1569245.186	-527409.429
	4	-641469.462	195552.244	.066	-1311909.426	28970.503
	5	-169182.539*	48620.921	.045	-335876.655	-2488.422
2	1	2918242.308*	261972.459	.000	2020084.339	3816400.277
	3	1869915.000*	396128.186	.005	511811.601	3228018.399
	4	2276772.846*	391247.475	.001	935402.694	3618142.998
	5	2749059.769*	268819.729	.000	1827426.319	3670693.220
3	1	1048327.308*	151940.018	.000	527409.429	1569245.186
	2	-1869915.00*	396128.186	.005	-3228018.399	-511811.601
	4	406857.846	179666.333	.429	-209118.158	1022833.851
	5	879144.769*	136418.798	.000	411440.526	1346849.012
4	1	641469.462	195552.244	.066	-28970.503	1311909.426
	2	-2276772.85*	391247.475	.001	-3618142.998	-935402.694
	3	-406857.846	179666.333	.429	-1022833.851	209118.158
	5	472286.923	182423.377	.237	-153141.452	1097715.298
5	1	169182.539*	48620.921	.045	2488.422	335876.655
	2	-2749059.77*	268819.729	.000	-3670693.220	-1827426.319
	3	-879144.769*	136418.798	.000	-1346849.012	-411440.526
	4	-472286.923	182423.377	.237	-1097715.298	153141.452

Based on estimated marginal means

*. The mean difference is significant at the ,05 level.

b. Adjustment for multiple comparisons: Bonferroni.

Figure 5.
SPSS Output for Pairwise Comparisons

Table 4.
Descriptive TIC Ranking and Bonferroni-Adjusted Pairwise Results (significance at a = .05)

Rank	Method	Note
1	Wagner–Whitin (WW)	WW significantly lower than L4L, EOQ, and PPB; not significantly different from POQ.
2	Part-Period Balancing (PPB)	PPB significantly lower than L4L and EOQ; significantly higher than WW; not significantly different from POQ.
3	Periodic Order Quantity (POQ)	POQ significantly lower than L4L and EOQ; not significantly different from WW and PPB.
4	Economic Order Quantity (EOQ)	EOQ significantly lower than L4L; but significantly higher than WW and PPB; not significantly different from POQ.
5	Lot-for-Lot (L4L)	Highest TIC; significantly less efficient than all other methods.

Sumber: Summary from SPSS output for pairwise comparisons

Comparative Discussion of Lot-Sizing Performance
The relatively low TIC generated by WW can be explained by its full-horizon replenishment logic. Rather than evaluating each period independently, WW compares alternative combinations of order timing and demand coverage across the planning horizon. In the present case, each order incurred a fixed cost of IDR 550,000, whereas the holding cost was IDR 1,097 per unit. This relationship made frequent replenishment comparatively expensive and favored the consolidation of requirements when the ordering-cost savings exceeded the additional holding cost.

This result supports the view that WW is particularly effective under deterministic but time-varying demand when ordering and holding costs can be evaluated across the full horizon. It is consistent with Adipradana and Muharni (2021), who found that WW could produce substantial inventory-cost savings. However, the result also reinforces the observation of Asmal et al. (2023) that WW is not necessarily superior for every item or demand pattern. Its performance remains dependent on the interaction between demand variation, ordering costs, and holding costs.

PPB generated the second-lowest aggregate TIC, only approximately 5.47% above WW. Its relatively strong performance can be attributed to its effort to balance the cost of placing an additional order against the holding cost created by combining requirements. Although PPB did not reproduce the complete dynamic optimization of WW, it approximated the same economic trade-off through a simpler heuristic procedure. This result is consistent with Florim et al. (2019) and Maharani et al. (2025), who indicate that PPB can provide competitive or near-optimal outcomes under fluctuating demand. Nevertheless, the pairwise result shows that PPB remained statistically more costly than WW in the present dataset.

L4L produced the highest TIC because it generated replenishment decisions that closely followed each period's net requirement. This approach reduced inventory carried between periods but resulted in frequent orders and repeated ordering costs. Under the cost structure of IM Workshop, the savings in holding cost were not sufficient to offset the cost of placing many individual orders. The finding is consistent with Ventura and Lu (2023), who note that L4L becomes less attractive when setup or ordering costs are high relative to holding costs.

The L4L result differs from Baroroh et al. (2024) and Saptadi et al. (2023), who found L4L to be a relatively low-cost alternative in their respective applications. This difference suggests that the performance of L4L is context dependent. It may perform well when minimizing inventory holdings is particularly important or when frequent ordering does not create a substantial cost penalty. In the present workshop case, the relatively high fixed ordering cost favored methods that consolidated purchases rather than matching every period's demand separately.

EOQ and POQ occupied intermediate positions. EOQ determines an economically efficient order quantity from aggregate demand and cost information, but it is less responsive to period-specific changes than WW. POQ translates EOQ logic into replenishment intervals and can therefore provide a more regular ordering pattern. The absence of a statistically significant difference between EOQ and POQ reflects their related economic foundations. Their intermediate performance is consistent with Nurprihatin et al. (2022), Arif and Sukarno (2021), and Kurniawan and Raphaeli (2018), whose findings indicate that the relative effectiveness of EOQ and POQ depends on item characteristics and demand patterns. Overall, the TIC results do not indicate that one classical method is universally superior. WW was the most cost-efficient method for the present demand and cost structure, while PPB provided the closest aggregate alternative. EOQ and POQ offered simpler but less cost-efficient rules, and L4L incurred a substantial cost penalty because of its frequent replenishment pattern. These differences provide the operational-cost foundation for the subsequent evaluation of monthly purchasing expenditures and financial exposure.

Monthly Purchasing Expenditure and Cash-Ceiling Evaluation

Although TIC measures the ordering and holding costs generated by each lot-sizing method, it does not directly represent the amount of cash required to purchase inventory in a particular month. Therefore, the purchasing schedules were also converted into monthly purchasing expenditures by multiplying the scheduled order quantities by the corresponding unit purchase prices. These expenditures were evaluated separately from TIC because a method with a low inventory-related cost may still produce a concentrated cash requirement.

Table 5 presents the monthly purchasing expenditures generated by the five methods. The results reveal substantial differences in the timing and concentration of purchases. WW generated relatively large expenditures in January, March, May, July, September, and November, including a peak expenditure of approximately IDR 97.20 million in July. The months with no or relatively small purchases indicate that the method consolidated several periods of demand into selected replenishment points. This consolidation contributed to its low TIC but created substantial variation in monthly cash requirements. PPB exhibited a comparable consolidation pattern, although the timing of its purchases differed from WW. Large expenditures occurred in January, June, August, and November, with the highest expenditure reaching approximately IDR 111.06 million in August. POQ also generated concentrated purchases because its periodic replenishment cycles combined several monthly requirements. Its expenditures exceeded IDR 50 million in six months, including approximately IDR 115.81 million in January and IDR 95.85 million in May.

EOQ generated a less regular pattern because a predetermined economic order quantity was applied against changing monthly requirements. Large purchases were recorded in January, May, July, and December. Although its purchasing profile was less concentrated than that of POQ in cumulative terms, it still produced several months with substantial cash requirements.

In contrast, L4L distributed purchasing expenditures more evenly across the planning horizon because orders closely matched each month's requirements. Its monthly expenditures generally remained below the IDR 50 million base-case ceiling, with exceedances occurring only in August and December. However, this smoother purchasing profile was achieved through more frequent orders, which explains its higher TIC.

Under the base-case ceiling of IDR 50 million, WW recorded six violations and a cumulative excess of IDR 140.34 million. L4L recorded only two violations and a cumulative excess of IDR 4.70 million. EOQ generated five violations and cumulative excess of IDR 126.87 million, while POQ generated six violations and the highest cumulative excess, amounting to IDR 176.32 million. PPB recorded five violations and cumulative excess of IDR 137.88 million.

These findings demonstrate that the ranking based on TIC differs from the ranking based on monthly financial exposure. WW and PPB achieved relatively low TIC by consolidating purchases, but the same mechanism produced large cash requirements in selected months. L4L generated the highest TIC but the most manageable cash-expenditure profile. EOQ occupied an intermediate position, while POQ combined a moderate TIC with relatively high cash concentration. Because violation frequency and cumulative excess depend on the level of the purchasing ceiling, the robustness of these base-case findings was further examined through sensitivity analysis using IDR 40 million, IDR 50 million, and IDR 60 million ceilings. The results of this analysis and the corresponding policy recommendations are presented in the following subsection.

Table 5.
Monthly Purchasing Expenditures Generated by the Five Lot-Sizing Methods

Month	WW	L4L	EOQ	POQ	PPB
Jan	87.660.300	18.063.150	112.595.350	115.807.600	103.261.000
Feb	-	35.862.150	-	-	-
Mar	78.982.000	48.587.150	56.609.600	60.827.150	50.692.650
Apr	1.061.750	40.458.200	14.604.500	2.332.750	44.860.800
May	68.685.500	46.236.750	65.117.500	95.852.250	27.091.900
Jun	17.735.250	42.442.500	25.545.600	830.000	67.598.250
Jul	97.198.850	47.251.500	80.150.250	74.401.550	12.870.450
Aug	11.064.000	52.374.350	49.015.350	38.547.800	111.063.550
Sep	52.440.100	46.424.750	38.546.000	68.069.050	10.475.500
Oct	35.045.000	37.223.150	24.665.100	-	17.838.600
Nov	55.376.950	39.482.950	38.343.500	61.363.950	55.266.200
Dec	1.487.200	52.330.300	62.394.100	6.826.500	5.718.000

Sumber: Author's calculation based on POM for windows simulation and IM Workshop operational data (2024)

Managerial Interpretation and Sensitivity Analysis

Table 6 presents the performance of the five lot-sizing methods under monthly purchasing ceilings of IDR 40 million, IDR 50 million, and IDR 60 million. The IDR 50 million ceiling represents the base-case scenario, while the lower and higher ceilings represent tighter and more flexible liquidity conditions, respectively. TIC remains unchanged across the three scenarios because the purchasing ceiling is applied only after the original lot-sizing schedules have been generated. Changes in the ceiling therefore affect violation frequency, cumulative excess, and the resulting policy recommendations rather than the inventory-cost outcomes themselves.

Table 6.
Sensitivity of Lot-Sizing Performance to Alternative Monthly Purchasing Ceilings

Monthly ceiling	Method	TIC (Rp)	Violations	Excess (Rp)
40 million	WW	40.222.088	6	200.343.700
	L4L	78.159.238	8	56.105.500
	EOQ	53.850.343	6	185.882.150
	POQ	48.561.191	6	236.321.550
	PPB	42.421.461	6	192.742.450
50 million	WW	40.222.088	6	140.343.700
	L4L	78.159.238	2	4.704.650
	EOQ	53.850.343	5	126.866.800
	POQ	48.561.191	6	176.321.550
	PPB	42.421.461	5	137.881.650
60 million	WW	40.222.088	4	92.526.650
	L4L	78.159.238	0	0
	EOQ	53.850.343	4	80.257.200
	POQ	48.561.191	6	116.321.550
	PPB	42.421.461	3	101.922.800

Sumber: Author's simulation and decision analysis based on policy evaluation under cashflow constraints (IM Workshop, 2024)

Table 7.
Policy Recommendations under Alternative Monthly Purchasing Ceilings

Monthly ceiling	Cost-first policy	Cash-first policy	Balanced policy
40 million	WW	L4L	No feasible method
50 million	WW	L4L	EOQ
60 million	WW	L4L	WW

Sumber: Author's simulation and decision analysis based on policy evaluation under cashflow constraints (IM Workshop, 2024)

As the monthly ceiling increases, most methods show lower violation frequencies and cumulative excess values. L4L exhibits the lowest liquidity exposure under all three scenarios. At the IDR 40 million ceiling, L4L records eight violations but generates the smallest cumulative excess, indicating that its purchasing expenditures exceed the ceiling relatively frequently but only by comparatively small amounts. At the IDR 50 million ceiling, its violation frequency falls to two and its cumulative excess declines substantially. At the IDR 60 million ceiling, L4L produces no violations or cumulative excess. By contrast, WW, POQ, and PPB generate more concentrated purchasing expenditures, resulting in larger cumulative excess values, particularly under tighter liquidity conditions.

Based on these cost and liquidity profiles, Table 7 summarizes the methods selected under the three managerial policy orientations. The cost-first policy consistently selects WW because it generates the lowest TIC, and this result is unaffected by changes in the purchasing ceiling. The cash-first policy consistently selects L4L because it produces the lowest cumulative excess under all three scenarios.

The balanced policy is more sensitive to the firm's liquidity condition. Under the IDR 40 million ceiling, none of the five methods satisfies all predefined limits for TIC, violation frequency, and cumulative excess. This result indicates that under severe liquidity constraints, the firm may need to revise the tolerance limits, obtain additional working capital, or adjust the replenishment schedules before a balanced alternative can be selected. Under the base-case ceiling of IDR 50 million, EOQ is selected because it satisfies the balanced-policy limits while generating the lowest TIC among the eligible alternatives. When the ceiling increases to IDR 60 million, WW becomes eligible and is selected because it produces the lowest TIC among the methods satisfying the financial-exposure limits.

These findings indicate that the cost-first and cash-first recommendations are relatively robust across alternative liquidity scenarios, whereas the balanced recommendation changes as financial flexibility increases. This sensitivity is consistent with the purpose of the balanced policy, which is intended to adjust the trade-off between inventory-cost efficiency and liquidity exposure according to the firm's working-capital condition.

Theoretical Implications

This study provides several implications for inventory management and lot-sizing research. First, it extends the conventional evaluation of classical lot-sizing methods beyond Total Inventory Cost. Traditional comparisons commonly rank replenishment methods according to the trade-off between ordering and holding costs. The present findings show that this ranking is not necessarily equivalent to the ranking based on periodic financial exposure. WW and PPB generated relatively low TIC by consolidating replenishment requirements, but the same mechanism created large purchasing expenditures in selected periods. Conversely, L4L generated a higher TIC while producing a more manageable expenditure profile. This distinction indicates that inventory-cost efficiency and purchasing-liquidity exposure represent related but analytically different dimensions of lot-sizing performance.

Second, the study connects two streams of inventory research that are often examined separately. Classical deterministic lot-sizing studies generally focus on cost-efficient replenishment schedules, whereas cash-constrained inventory studies embed cash positions, borrowing, or financing mechanisms directly into dynamic optimization models. The present study introduces an intermediate evaluation approach: established lot-sizing procedures are retained to generate purchasing schedules, and their outputs are subsequently assessed through a financial-feasibility benchmark. This two-stage structure demonstrates how financial considerations can be incorporated into the evaluation of classical methods without modifying their original calculation procedures.

Third, the study operationalizes liquidity exposure through violation frequency and cumulative excess. Violation frequency captures how often a purchasing schedule exceeds a recurring financial benchmark, whereas cumulative excess captures the magnitude of the resulting exposure. The use of both indicators shows that financial feasibility cannot be represented adequately by a single binary distinction between feasible and infeasible schedules. A method may exceed the ceiling frequently by relatively small amounts or less frequently by substantially larger amounts, creating different working-capital implications.

Fourth, the sensitivity analysis demonstrates that the preferred lot-sizing method is contingent on the managerial objective and the firm's financial flexibility. The cost-first and cash-first recommendations remained stable across the three purchasing-ceiling scenarios, whereas the balanced recommendation changed as the ceiling increased. This finding supports a contingent view of lot-sizing selection: no method is universally superior because the preferred alternative depends on the interaction between inventory-cost objectives, liquidity exposure, and working-capital conditions.

Accordingly, the theoretical contribution of this study lies not in proposing a new lot-sizing algorithm, but in broadening the conceptual basis on which established methods are compared and selected. The resulting liquidity-aware evaluation framework complements traditional inventory-cost analysis and helps bridge classical lot-sizing research with the operations-finance perspective relevant to financially constrained SMEs.

Conclusion

The findings show that no single lot-sizing method is universally superior. WW achieved the lowest aggregate TIC, followed closely by PPB, whereas L4L generated the highest inventory-related cost because of its frequent replenishment pattern. EOQ and POQ occupied intermediate positions, reflecting different compromises between order consolidation, ordering frequency, and inventory carried across periods. These differences confirm that the economic performance of a lot-sizing method is shaped by the interaction between demand variation, ordering costs, holding costs, and the replenishment logic embedded in each method.

The financial-feasibility evaluation revealed that the ranking based on TIC was not equivalent to the ranking based on liquidity exposure. Methods that consolidated demand across periods, particularly WW and PPB, achieved relatively low TIC but generated concentrated purchasing expenditures in selected months. L4L produced the opposite profile: its frequent and smaller replenishments increased TIC but resulted in the lowest cumulative excess above the monthly purchasing ceiling. Under the IDR 50 million base-case scenario, the cost-first policy therefore selected WW, the cash-first policy selected L4L, and the balanced policy selected EOQ as the lowest-cost method satisfying the predefined TIC, violation-frequency, and cumulative-excess limits.

The sensitivity analysis further demonstrated that the preferred method depends on the firm's financial flexibility and managerial priority. The cost-first recommendation remained unchanged across the IDR 40 million, IDR 50 million, and IDR 60 million scenarios because TIC was not affected by the subsequent purchasing-ceiling evaluation. The cash-first policy also consistently selected L4L because it generated the lowest cumulative excess under all three scenarios. The balanced recommendation was more responsive to liquidity conditions: no method satisfied all balanced-policy thresholds under the IDR 40 million ceiling, EOQ was selected under the IDR 50 million base case, and WW became the preferred balanced alternative when the ceiling increased to IDR 60 million. This result indicates that the balanced policy functions as an adaptive selection mechanism rather than producing a single recommendation for every financial condition.

From a theoretical perspective, this study extends classical lot-sizing evaluation by distinguishing inventory-cost efficiency from purchasing-liquidity exposure. Traditional comparisons generally assess methods through ordering and holding costs, whereas the present findings show that a cost-efficient replenishment schedule may still create a difficult working-capital profile when purchases are concentrated in particular periods. The study therefore provides a conceptual link between deterministic lot-sizing analysis and the operations–finance perspective through a two-stage approach: the established methods are first used to generate replenishment schedules, after which their outputs are evaluated against recurring financial benchmarks.

The study also operationalizes liquidity exposure through violation frequency and cumulative excess. These measures capture two different characteristics of financial exposure: how often the purchasing ceiling is exceeded and by how much the total expenditure exceeds that ceiling. Their combined use provides a more informative evaluation than a simple feasible–infeasible classification. Accordingly, the contribution of this study does not lie in proposing a new cash-constrained lot-sizing algorithm, but in providing a liquidity-aware framework for comparing and selecting established methods according to cost-first, cash-first, and balanced managerial orientations.

For managers of financially constrained SMEs, the findings indicate that the selected method should reflect the firm's working-capital position and strategic priority. WW is appropriate when inventory-cost efficiency is the primary concern and sufficient liquidity or purchasing flexibility is available. L4L is more suitable when reducing concentrated cash exposure is prioritized, despite its higher inventory-related cost.

The balanced framework allows managers to select the lowest-cost alternative that remains within explicitly defined financial-exposure tolerances. The specific thresholds should therefore be adapted to the firm's available working capital, supplier-credit conditions, risk tolerance, and capacity to split or reschedule orders.

Several limitations should be acknowledged. The analysis was based on one workshop, one product category, and a deterministic one-year demand dataset. The model did not incorporate demand uncertainty, lead-time variability, stockout costs, service-level performance, quantity discounts, supplier credit, or dynamic cash inflows. The monthly purchasing ceilings and balanced-policy thresholds were also case-specific analytical benchmarks rather than universally optimal values. Consequently, the numerical recommendations should not be generalized without considering the operational and financial characteristics of other firms.

Future research may extend the framework by incorporating demand forecasting, stochastic demand, uncertain lead times, service-level measures, and financing arrangements such as trade credit or short-term borrowing. The model may also be applied to multiple product categories and different SME sectors to evaluate its external validity. A further extension could embed inventory-cost, service-level, and liquidity indicators within a multi-objective or stochastic optimization framework. Such developments would enable SMEs to move from retrospective policy evaluation toward more adaptive and forward-looking inventory planning.

Declaration

Author Contribution

All authors contributed equally as the main contributors of this paper. All authors read and approved the final paper.

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Competing Interest

The authors declare that they have no conflicts of interest to report regarding the present study.

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