JOURNAL OF
BUSINESS AND MANAGEMENT
Vol. 3, No.7, 2014: 737-751

OPTIMAL PORTFOLIO ANALYSIS WITH RISK-FREE ASSETS USING INDEX-TRACKING AND MARKOWITZ MEAN-VARIANCE PORTFOLIO OPTIMIZATION MODEL

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Abstract- Constructing optimal portfolio to the desired expected return is one of the main concerns of every investor. This research aimed for constructing and comparing two approaches of stocks portfolio optimization model with an addition of risk-free assets on two different models. The classical Markowitz mean-variance model is further compared with an index-tracking model introduced by Edirisinghe. Additional risk-free asset in the portfolio is intended to give investors an option to lower the risk through diversification. The stocks analyzed for the research are stocks traded in Jakarta Composite Index (JCI) under the period of 2007 – 2012. Results shown that additional risk-free asset lowers the risk significantly for both Markowitz and the index-tracking portfolios, with the index-tracking diversified portfolio has a lower risk than the benchmark index. The index-tracking portfolio also gives a higher beta than the Markowitz MV portfolio. This increase in beta depends on the index variance, in this case JKSE variance, and also the asset covariance matrix. During the back testing, the performance of both Markowitz MV portfolio and index-tracking portfolio do not track the index performance. However, the portfolios which use index-tracking method outperform the portfolios constructed using the Markowitz MV model.

Keywords: Investment, Index-tracking, Portfolio Optimization, Mean-variance Optimization

Introduction

While investing, an investor or a portfolio manager wishes for owning an optimum portfolio to be invested. Either by having the maximum return with a certain level of risk, or by minimizing the risk of the desired mean return. There is a tendency to choose the small amount of stocks other than choosing many stocks, for it can reduce the transaction and administration costs, as well as preventing the portfolio for having illiquid positions (Jansen and Van Dijk, 2002). The classical Markowitz mean variance theory stresses on the idea that an investment can be diversified and thus resulting in lower risk for a certain level of mean return. However, the model has a flaw in terms of the reliability of the portfolio to be implemented to the real investment, since the portfolio uses the historical data for the portfolio optimization.

Another method an investor might use is the benchmark index-tracking method. Portfolio managers who use the benchmark index-tracking approach as one of the ways to invest the assets find the portfolio sometimes stresses on the excess returns that it forgets the increasing risk as the returns increases (Jorion, 2003). There is a possibility to reduce the risk of an index-tracking portfolio by adding the risk-free assets in the portfolio. However, it raises a problem on the impact of the risk-free assets within the portfolio. The risk-free assets reduce the risk, but the return decreases as the risk-free assets have a relatively low return.

The author is striving to conclude and combine the issues and topics mentioned above to construct a proper analysis of the impact of applying two different methods of portfolio optimization to the portfolio performance, and further searches for the effect of adding the risk-free asset to the portfolio risk. The Jakarta Composite Index (IHSG) is used for the benchmark and the stocks selection. As for the risk-free assets, the arithmetic monthly average BI Rate is used for the calculation. Several assumptions are made during the research, which are that the data is distributed normally and there is no transaction costs, taxes, and other financial costs related with the investment choices.

Literature Review

Investment

According to the first chapter of Investments, investment is defined as follows: *"An investment is the current commitment of money or other resources in the expectation of reaping future benefits"* (Bodie *et al.,* 2009:1). In brief, investment is an asset that is acquired in order to foresee income in the future. It is a monetary asset an investor puts for a certain period of time, and after certain period of time it is expected that the asset will generate income or getting appreciated and sold with some gain. For example, an individual purchases stocks and hold it for several years, with the expectation of increasing stock prices and the dividends given by the companies, with the possibility of having risk on the stocks investments.

Portfolio Theory

Jones (2014:4) stated that Portfolio is the securities held by an investor taken as a unit. Another definition by Bodie et. al. explains portfolio as "an investor's collection of investment assets" (2009:9). Since there is a risk-return trade-off while investing in particular assets, constructing a diversified portfolio consisting of different types of assets can be one choice to reduce the risk borne by one particular asset. Diversification itself means the process to hold many assets within the portfolio in order to limit one exposure of any particular asset (Bodie et. al., 2009:11).

In order to cope with the dynamic changes of asset risk and return, an investment manager should be able to manage the portfolio periodically. Portfolio management is *"a process of combining securities in a portfolio tailored to the investor's preference and needs, monitoring that portfolio, and evaluating its performance"* (Bodie et. al., 2009:G-9). In brief, an investment manager or any investor who holds a portfolio needs to analyze and select the securities, keeping an eye on it, and further assessing the portfolio due to changes in the market condition.

Markowitz Portfolio Theory

In 1952, Harry Markowitz introduces the foundation of Modern Portfolio Theory ("MPT"). It is noted that his contribution to the economics is his analysis on the impact of diversifying the securities within a portfolio and its covariance relationships (Mangram, 2013:60). Markowitz portfolio theory stresses on analyzing the performance of a given portfolio based on the means and the variance of the return of the assets contained in the portfolio, with the assumptions of a risk-averse investor, who is willing to get a small risk on a certain level of expected return (Marling & Emanuelsson, 2012:2).

According to Edirisinghe (2013), the Markowitz portfolio optimization model is as follows,

$$\begin{aligned} &\min_{x} \ \frac{1}{2}x'Vx \\ &\text{s.t.} \ \mu' x = m \\ &\quad \mathbf{1}'x = \mathbf{1} \end{aligned}$$

Equation 6: Average Collection Period

Given that x is the weight of each asset, V is the variance-covariance matrix, μ is the asset mean return, and m is the specified mean return of the portfolio.

In order to construct an optimum portfolio of current asset, a proper risk-adjusted performance needs to obtained by adjusting the composition of each asset. The Markowitz MV model shows the trade-offs between risk and return of an asset, shown by the changes in expected return for a level of variance, called the efficient frontier. Efficient frontier is one that has the smallest portfolio risk for a given level of expected return or the largest expected return for a given level of risk (Na, 2008:2).

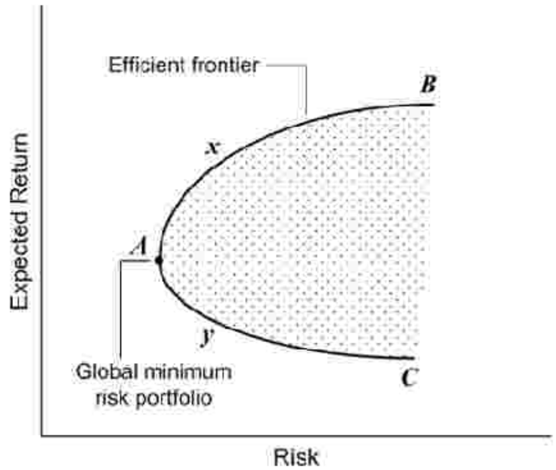

Figure 1: Markowitz Efficient Frontier

Based on the figure shown above, the global minimum-risk portfolio is indicated by point A, meaning that the portfolio has the lowest risk. The minimum risk-frontier AC is the bottom segment which is dominated by the AB portfolio, since the AB portfolio frontier has bigger return than the AC.

Index-tracking Portfolio Theory

As a development of Markowitz Mean-Variance model, Edirisinghe (2013) first introduced the index-tracking optimal portfolio selection with the basic idea of constructing a portfolio which is holding relatively few stocks compared to investing in many stocks, which results in very small and illiquid positions (Jansen and van Dijk, 2002:33). The model gives an extension of the Markowitz mean-variance model. In this model, the classical risk measure of portfolio variance is replaced with the variance of the tracking error between the index return and the return on the portfolio of n risky assets. Suppose a portfolio *P* consists of the risky assets from the index *M*. Its main objective is for a return of a portfolio, denoted by r_P, to closely 'track' the benchmark index of a portfolio, denoted by r_M. When the Var($r_P - r_M$) is the lowest possible for a desired level of expected return of a portfolio, the univariate random variables of r_P and r_M is in-synch in terms of direction and the magnitude (Edirisinghe, 2013:16). Denote the return of asset *j* be r_j, and the portfolio weight in asset *j* be x_j, and the portfolio return is $r_P = \sum_j x_j r_j$. The equation for the indicator of portfolio *P* tracking the index *M*, the Var($r_P - r_M$), is further explained by Edirisinghe as follows,

$$\begin{aligned} Var(r_P - r_M) &= \sigma_P^2 + \sigma_M^2 - 2Cov(r_P, r_M) \\ &= \sigma_P^2 + \sigma_M^2 - 2\sum_j x_j \sigma_{jM} \\ &\quad (\text{since } r_P = \sum_j x_j r_j) \\ &= \sigma_P^2 + \sigma_M^2 - 2\sigma_M^2 \beta_P \end{aligned}$$

Equation 6: Average Collection Period

given that $Var(r_P - r_M)$ = variance of the difference between r_P and r_M, σ_P^2 is the variance of portfolio *P*, σ_M^2 =variance of the index *M*, $Cov(r_P, r_M)$ is the covariance between r_P and r_M, and β_P = beta portfolio *P.* With the desired threshold on portfolio mean return, *m*, the equation of the index-tracking objective is represented as follows,

$$\min_x \frac{1}{2} x'Vx - \sigma_M^2 \beta' x$$
$$\text{s.t. } \mu' x = m$$
$$\mathbf{1}' x = \mathbf{1}$$

Equation 6: Average Collection Period

given that *x* = weight of each asset, *V* = variance-covariance matrix, μ is the asset mean return, *m* = specified mean return of the portfolio, and $\beta' x$ is the portfolio beta.

Index-tracking Portfolio with Risk-free Assets

The index-tracking portfolio with all risky assets offers a portfolio which can track the benchmark index. However, some problems regarding the index-tracking portfolio is that one mainly focuses on the return that the risk is higher than using the classical Markowitz MV method (Jorion, 2003). Thus, additional risk-free assets on the portfolio can be an option to reduce the risk borne by the all-risky assets portfolio. With the desired threshold on portfolio mean return, *m*, the equation of the index-tracking objective is represented as follows

$$\min_x \frac{1}{2} x'Vx - \sigma_M^2 \beta' x$$
$$\text{s.t. } \quad \mu' x + (\mathbf{1} - x'\mathbf{1}) r_f = m$$

Equation 6: Average Collection Period

given that *x* = weight of each asset, *V* = variance- covariance matrix, μ = asset mean return, *m* is the specified mean return of the portfolio, $\beta' x$ = portfolio beta, and = risk-free assets.

Back-testing Theory

A common problem of applying one method to another while constructing the portfolio is the fact that historical data used for obtaining the portfolio sometimes does not represent the actual return in the future. Thus, a test needs to be conducted in order to test whether the portfolio performs well just as the prediction by using the simple back test. Back testing is, in brief, a way to test a trading strategy, specifically the stocks portfolio, within a certain period of time, see Inoue and Killian (2002). Rather than applying the strategy in the real time, an investor can do the back testing by having a simulation of the previous strategy on the different but relevant past data.

Portfolio Performance Evaluation

In order to know whether a portfolio has a good performance or not, there are several tests which can be done to it. The tests aim for indicating whether the portfolio has a proper return within the range of acceptable risk. The tests could also indicate whether a portfolio outperforms the market in terms of risk or return. Commonly there are three measurement tests for the portfolio performance, which are the Shape's Ratio Measure, Jensen's Alpha Ratio, and the Treynor Measure.

Sharpe's Ratio Measure

According to Bodie *et. al.,* Sharpe's ratio *"measures the reward-to-volatility ratio of portfolio excess return or risk premium to standard deviation"* (Bodie *et.* al 2009:129). The ratio measures the risk-adjusted performance, identifying how much return one would gain in exchange for a certain level of risk. The greater the value of Sharpe's ratio, the better its risk-adjusted performance, since it results in a bigger risk premium per unit of deviation. The equation of Sharpe's ratio is shown below,

$$Sharpe\ Ratio = \frac{\bar{r}_p - r_f}{\sigma_p}$$

Equation 6: Average Collection Period

given that $\bar{r}_p$ = Average return of portfolio *p*, r_f = Risk-free rate, σ_p =Standard deviation of portfolio *p.*

Jensen's Alpha Measure

The theory of Jensen's Alpha drives from the capital asset pricing model (CAPM). This ratio measures the average return on the portfolio predicted by using the CAPM, given the portfolio beta and the average market return is obtained. It results in the alpha of the investment (Bodie *et. al* 2009: 826). A positive alpha indicates a better risk-adjusted return of the portfolio, and that the portfolio outperforms the market by earning excess return from the risk premium. In order to calculate the alpha of a portfolio, the equation is represented as follows,

$$\alpha_p = \bar{r}_p - [R_f + (r_m - R_f)\beta_{pm}$$

Equation 6: Average Collection Period

given that α_p is the alpha of portfolio *p*, $\bar{r}_p$ is the expected return of portfolio *p*, r_m is the market return, R_f is the risk-free rate, and β_{pm} is the beta of portfolio *p* relative to the market.

Treynor Measure

"*Treynor's measure gives excess return per unit of risk, but it uses systematic risk instead of total risk*" (Bodie *et. al* 2011:826). Instead of the standard deviation, the Treynor's measure uses the beta of a portfolio relative to its market. The equation is represented as follows,

$$Treynor\ Measure = \frac{\bar{r}_p - r_f}{\beta_{pm}}$$

Equation 6: Average Collection Period

given that $\bar{r}_p$ is the expected return of portfolio *p*, R_f is the risk-free rate, and β_{pm} is the beta of portfolio *p* relative to its market.

Methodology

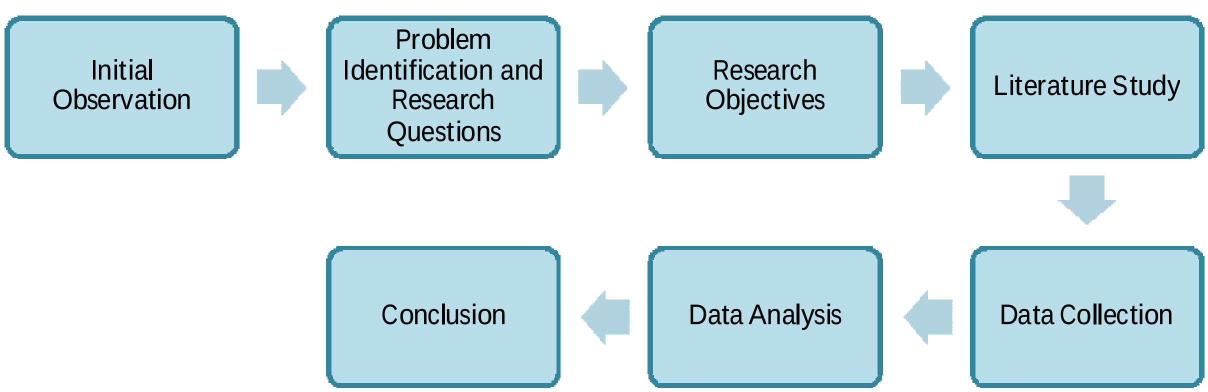

Figure 2: Research Methodology

Initial Observation: the observation is conducted from obtaining recent issues in the investment world and by looking at new methodologies and research related with investments, especially among stocks and risk-free investments.
Problem Identification and Research Questions: from the early observation, several problems are generated and the research questions are listed.
Research Objectives: the main objective of the research is to construct an optimal portfolio from the stocks and risk-free asset by using the index-tracking model, and to compare the portfolio performance and results with the Markowitz MV model.
Literature Study: a deeper literature study is conducted to fully understand the methodology and the model used during the research process.
Data Collection: The data gathering process that related with the main research topic
Data Analysis: The analyzed and calculated data as the core of this research
Conclusion: Upon conclusion the summary of all the research process and analysis is constructed.

Data Collection & Analysis

In order to analyze the data and test the model, data collection is conducted for the first analysis phase. According to the Indonesia Stock Exchange, there are 31 stocks which become the top-20 market capitalization during the period of 2007 – 2013. The stocks price used for the data is the monthly-adjusted closing price of the stock, considering the dividend payment, stock split, and other company transactions during the period of observation. The risk-free asset used in this project is obtained from the Indonesia central government bank website, which is the Bank Indonesia interest rate per annum, issued on the Bank Indonesia website during the period of 2007 – 2014. The risk-free rate used is the average of the BI rate during period of observation, which is 7.12% per annum.

Stocks Selection

Among the 31 stocks which had become the top-20 list for Indonesia market capitalization, there are eleven stocks which remained the top-20 list for Indonesia market capitalization during the period of 2007 – 2013. Before the stocks are used during the presentation, each stock performance is evaluated. Using the Jensen's measure of alpha ratio, there are six stocks which have positive alpha, indicating good performance of the stocks. The six stocks are then used for further analysis. The stocks and each alpha are presented on the table 1 below,

Table 1. Stocks and Alpha Calculation

Stocks	Alpha
ASII	0.070
UNTR	0.042
BMRI	0.026
PTBA	0.019
BBRI	0.019
INTP	0.011
PGAS	-0.008
BBCA	-0.009
HMSP	-0.022
TLKM	-0.033
UNVR	-0.043

Risk and Return of Individual Stock

The monthly return of the stock is calculated first by the logarithmic single period return, and after that the monthly average return is obtained from the calculation of arithmetic multi period average return. After conducting the calculation using Microsoft Excel, Figure 3 presents the average monthly risk and return of each stock, and the risk and return of the Jakarta composite index as the comparison from the period of January 2007 – December 2012.

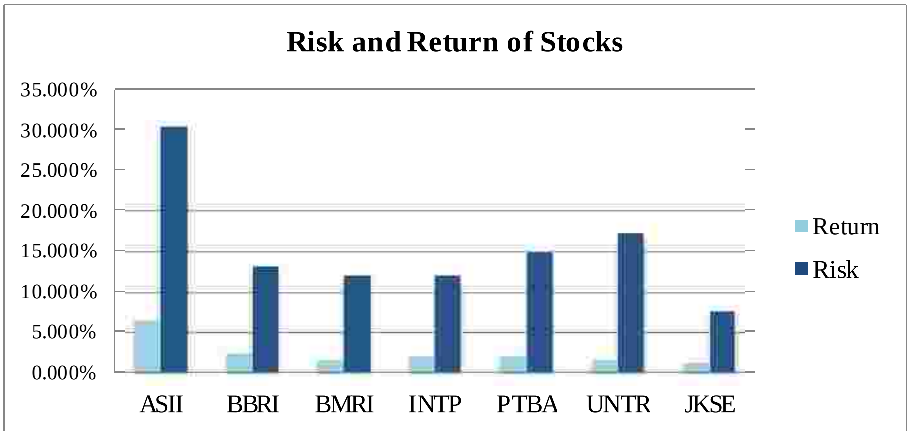

Figure 3: Risk and Return of Stocks

From the table above it can be shown that the average monthly return of Jakarta composite index is r_M =1.211% and the average monthly risk is s_M = 7.691%. Stock with the highest return is ASII with r_{ASII} = 6.466%. However, the risk is also the highest among all, which is s_{ASII} = 30.442%, unlike the other stocks which have the return between 1 – 3 % and the risk between 11 – 18%. Stock with the lowest return is BMRI with r_{BMRI} = 1.438%, while stock with the lowest risk is INTP with the value of s_{INTP} = 11.942%. Despite the high risk while investing in ASII, this stock is not as volatile as other stocks because of the high return provided during the period of research. Instead, the most volatile stock is UNTR which has a risk of s_{UNTR} = 17.141% but only give 1.522% gain in return. The 2008 global crisis also affected the stock prices to drop and the full recovery of the crisis had started in the early 2009. This also affects the high risk shown on the figure above.

Index-tracking Portfolio Construction

After the stocks had been filtered, the optimal portfolio can be constructed by using the index-tracking optimization model. The monthly returns for six stocks (tickers), ASII, BBRI, BMRI, INTP, PTBA, and UNTR, denoted by j = 1, 2, 3, 4, 5, and 6. The monthly stocks return used for the research is from January 2007 – December 2012, and as the benchmark target for the tracking, the Jakarta Composite Index (JKSE) is used. As for the risk-free asset, the average BI rate of r_f = 0.593% per month is used for the calculation. The computation of the optimal portfolio is conducted using the Microsoft Office Excel solver. The solver is used to compute the weight proportion of each stock, the desired mean return, the standard deviation, and the index-tracking model objective. According to the index-tracking optimization model, the objective is set to minimize $Var\ (r_P - r_M)$ with the desired mean return of r_M = 1.211% (the mean return of JKSE). The proportion of the stocks without risk-free assets by using index-tracking model is as represented in figure 4.

According to the graph below, there is one stock to short, which is the ASII stock with the percentage of x(ASII) = -8.672%. The other stocks have long position, with the highest percentage is allocated to BMRI. The stocks proportion for each stock is x(ASII) = -8.672%, x(BBRI) = 1.961%, x(BMRI) = 67.969%, x(INTP) = 25.206%, x(UNTR) = 4.823%, respectively. The proportion is due to the desired portfolio mean return of r_P = 1.2111%, in which the mean return of BMRI (r_{BMRI} = 1.438%) is the closest one to the desired portfolio return. The standard deviation of the portfolio is s_{P1} = 11.409%, and the beta of the portfolio is β_{P1} = 1.33972.

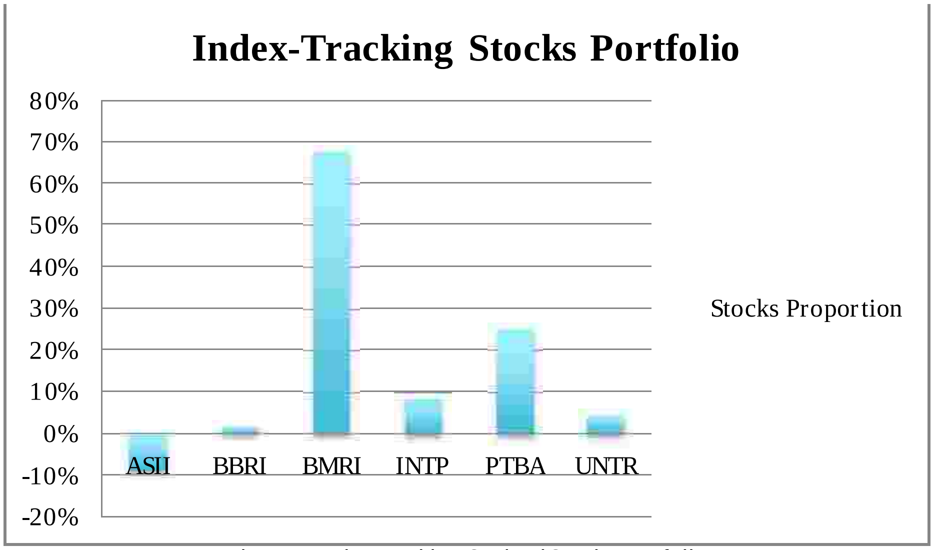

Figure 4: Index-tracking Optimal Stocks Portfolio

When a risk-free asset is added to the portfolio, there is a different composition of the portfolio generated due to the different constraints in portfolio weight. The desired mean return is still set to the mean return of JKSE of r_M = 1.2111% and the objective is set to minimize the *Var* $(r_P - r_M)$ but the portfolio return constraint has been changed to $u'x + (1\text{-}x'1)\, r_f = m$. The r_f is the monthly risk-free asset return, which is r_f = 0.593%. Figure 5 represents the composition of the index-tracking portfolio in addition of risk-free assets.

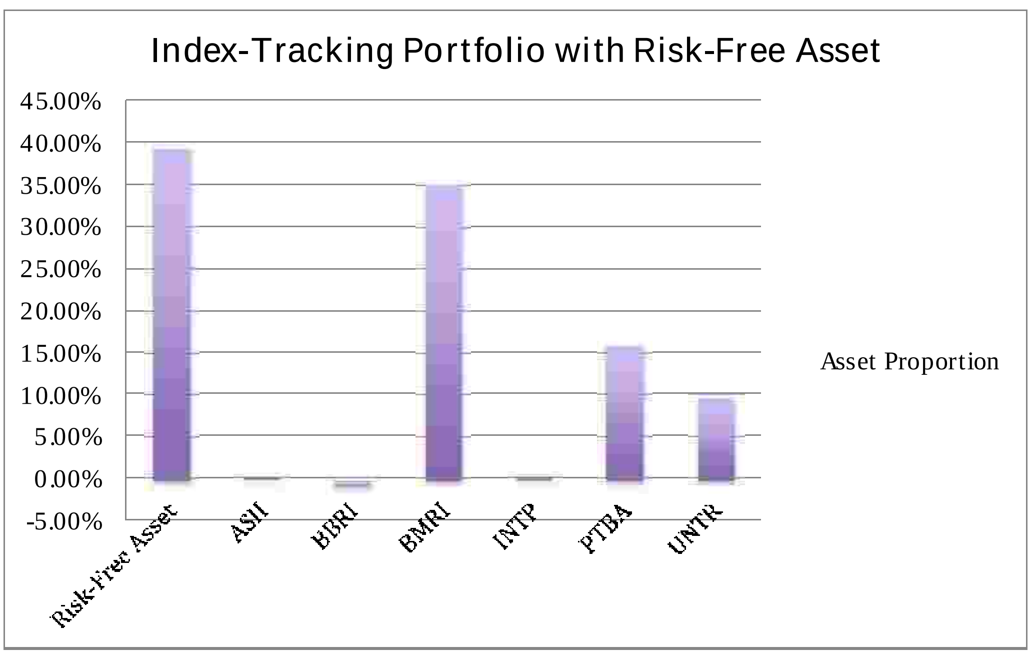

Figure 5: Index-Tracking Portfolio with Risk-Free Asset

With the same desired mean return, the index-tracking portfolio with addition of risk-free assets has most of the proportion allocated to the risk-free assets. The main objective is to minimize the *Var* $(r_P - r_M)$ of the stocks, and with addition of risk-free assets, it is possible to construct a portfolio which tracks the benchmark index because the remaining proportion can be allocated to the risk-free assets. Meanwhile, having the risk-free assets also minimize the risk and resulted in a more optimal portfolio. As indicated in this portfolio proportion, with the desired mean return of r_M = 1.2111%, the portfolio has standard deviation of s_{P2} = 7.0445%, even lower than the standard deviation of JKSE which is s_M = 7.6915%. The proportion for the assets are x(Risk-free Assets) = 39.434%, x(ASII) = 0.228%, x(BBRI) = -0.628%, x(BMRI) = 35.154%, x(INTP) = 0.276%, x(PTBA) = 15.920%, x(UNTR) = 9.616%, respectively. Below is the risk and return of the portfolio with different desired mean return.

Table 2: Portfolio Return and Risk

Portfolio Return	0.50%	1.00%	2.00%	3.00%	4.00%
s	6.5987%	6.7899%	8.7012%	11.7460%	15.2603%
Var (rp–rm)	0.0018251	0.0008195	0.0012570	0.0049596	0.0119272

Comparison with Markowitz Model

The objective of the index-tracking optimal portfolio is to minimize the *Var* $(r_P - r_M)$ while the classical Markowitz Mean Variance (MV) theory stresses on minimizing the risk of the portfolio with the desired mean return, or maximizing the Sharpe ratio. Therefore, the main concept is to create an optimal portfolio with the desired mean return which has the lowest unsystematic risk. In this research, with the same desired mean return of r_M = 1.211%, the optimal portfolio for both all-risky assets and mixed (risky and risk-free assets) are constructed. Figure 6 represents the optimal portfolio allocation based on the Markowitz MV theory.

Figure 6: Markowitz Stocks Portfolio

The MV portfolio has the standard deviation of s_{P3} = 10.9103%, with the portfolio mean return of r_{P3} = 1.211%. The portfolio has lower risk compared to the Index-tracking portfolio, which has the standard deviation of s_{P1} = 11.409%. This is due to the different set of objectives for both methods that result in different level of risk. The index-tracking portfolio takes into account the variance of JKSE in addition for the calculation, which results in additional risk from the JKSE itself. The MV portfolio has the stocks proportion allocated as x(ASII) = -13.3831%, x(BBRI) = 16.7697%, x(BMRI) = 56.4200%, x(INTP) = 30.9852%, x(PTBA) = 25.7401%, x(UNTR) = -16.5319%. There are two stocks to short in the Markowitz MV portfolio, compared with the index-tracking portfolio, which has only one stock to short. Edirisinghe (2013) mentioned earlier that under the lower threshold of risk, the index-tracking portfolio is more diversified than the Markowitz MV portfolio. The efficient frontier comparison as shown in figure 6 explains why the MV portfolio with the desired mean return of r_M = 1.211% has lower risk than the index-tracking portfolio.

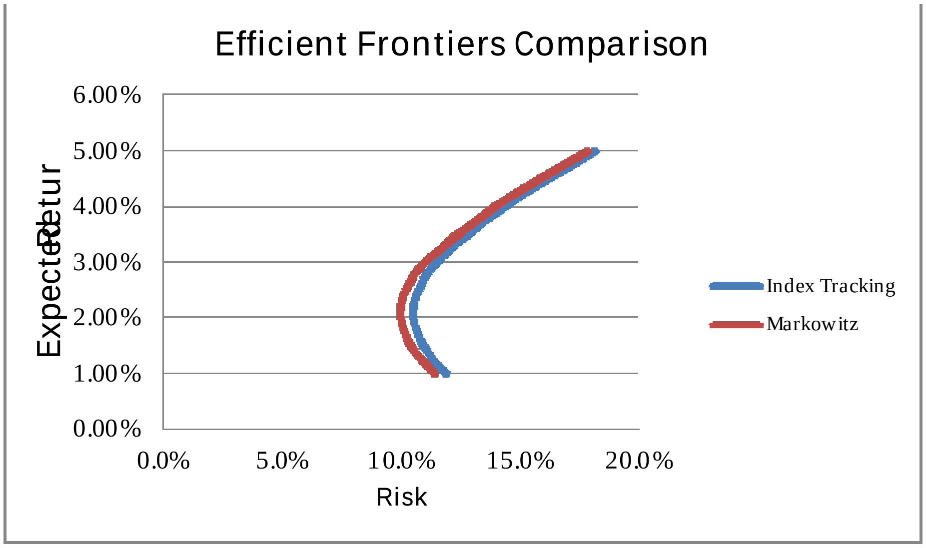

Figure 7: Efficient Frontiers Comparison

Based on the figure above, the index-tracking portfolio has higher risk in the same lower expected return. However, as the expected return increases, the index-tracking risk begins to coincide with the portfolio of the risk. In this research, the desired mean return of r_M = 1.211% is considered relatively low and according to the graph, the index-tracking risk of a portfolio with that level of mean return has higher risk than the Markowitz MV portfolio. However, the stocks are more diversified in the index-tracking portfolio allocation. Moreover, during the period of January 2007 – December 2012, the index-tracking portfolio performance tracks the benchmark index better than the MV portfolio. Both index-tracking and Markowitz MV portfolio have high risk when it comes to all-risky assets. The index-tracking portfolio has the standard deviation of s_{P1} = 11.409%, while the Markowitz MV portfolio has the standard deviation of s_{P3} = 10.9103%. These risks are still pretty high with the desired mean return of r_M = 1.211%, thus a lower level of risk is desired. By adding risk-free assets to the portfolio, there is a possibility that with the same desired mean return, the risk will be decreased. The index-tracking portfolio has shown that adding risky assets to the portfolio can reduce the standard deviation by 4.365%, which results in a lower risk. The portfolio of Markowitz with additional of risk-free assets is obtained to make a comparison with the previous index-tracking portfolio with additional risk-free assets. Using the same risk-free assets return of monthly average BI Rate return, r_f = 0.593%, the objective of minimizing the standard deviation with the desired mean return of r_M = 1.211%, the Markowitz portfolio with additional risk-free assets is constructed. Figure 8 shows the proportion of each asset allocated based on the Markowitz MV model.

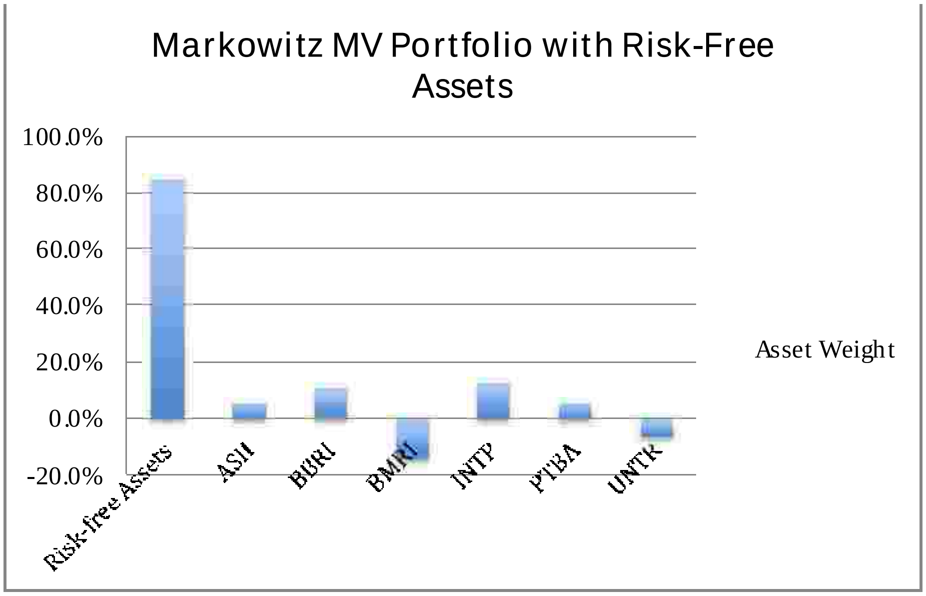

Figure 8: Markowitz MV Portfolio with Risk-Free Assets

It is clearly shown on the figure above that the risk-free assets has the biggest allocation of the portfolio with the proportion of *x*(Risk-Free Assets) = 84.7668%. This is because the main objective of the MV model is to minimize the risk, and since the risk-free assets has zero risk with the mean return of r_f = 0.593%, the solver allocates the proportion most on the risk-free assets. The proportion of each stock is *x*(ASII) = 5.7483%, *x*(BBRI) = 11.2048%, *x*(BMRI) = -14.1189%, *x*(INTP) = 12.8506%, *x*(PTBA) = 5.7781%, *x*(UNTR) = -6.2297%. Just as the MV portfolio of all-risky assets, the MV portfolio with additional risk-free assets has two stocks to short. Meanwhile, both index-tracking portfolios have only one stock to short. The Markowitz MV portfolio has the standard deviation of s_{P4} = 2.4947%, reducing the risk by 8.4156% from s_{P3} = 10.9103%. The risk is also lower than the risk bear by the index-tracking portfolio with additional risk-free assets, where the risk for the MV portfolio is s_{P4} = 2.4947% and the risk for the index-tracking portfolio is s_{P2} = 7.0445%. Having two diversified portfolios, the efficient frontier comparison can be presented as in Figure 9.

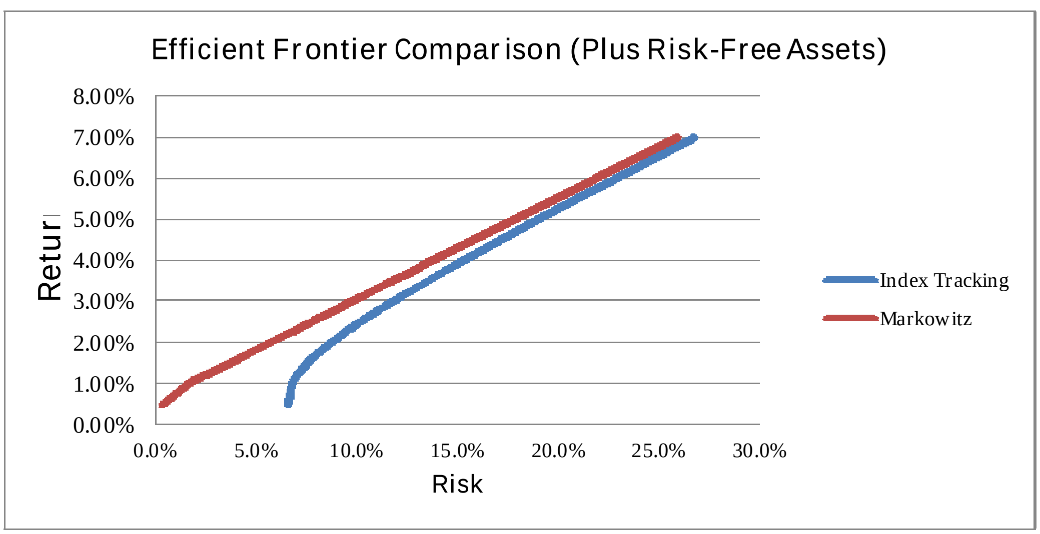

Figure 9: Efficient Frontier Comparison (Plus Risk-Free Assets)

Based on the figure above, it is shown that under lower level of return, the Markowitz MV model accommodates lower risk compared to the index-tracking portfolio. This is because the index-tracking portfolio focuses not only to minimize the risk, but also to track the benchmark index.

Therefore, the lowest risk will still be around the risk of the benchmark index, which in this case is the JKSE risk. When there is a risk-free assets added to the portfolio, at the lower level of risk the Markowitz MV model allocates most of the proportion to the risk-free assets, while the index-tracking model allocates the assets proportionally so that the stocks can track the benchmark index as close as possible. As the expected return increases, just as the all-risky assets portfolio, the risk coincides to the same value of standard deviation. When the historical stocks return performance of both index-tracking with risk-free assets and the Markowitz MV portfolio with risk-free assets are compared with the JKSE historical return, the index-tracking portfolio tracks the index return almost perfectly, giving a correlation of 0.9448, when the Markowitz MV portfolio does not track the index and has a correlation of 0.4060 with JKSE.

Back-testing The Portfolio

After constructing portfolios for risky assets and risk-free assets using both Markowitz MV model and index-tracking model, backtest of the model is required. Since there is no guarantee that the future returns will be similar with the past returns, backtesting the portfolio is needed. Testing the portfolio is done through creating portfolio performance from separated sets of data from different period of time. In this research, the time period used is 20% from the previous time period of the research. The period of the backtesting is done from January 2013 to April 2014. Theoretically, the performance of stocks during the period of the backtesting is not similar with the historical return during observation, and the index-tracking portfolio does not really track the performance returns anymore, since during the new period of time the index mean return and the standard deviation have changed. However, the index-tracking portfolio will track the benchmark index better than the Markowitz MV portfolio. Figure 10 and 11 represent the backtest result of stocks performance with difference allocation of risky assets.

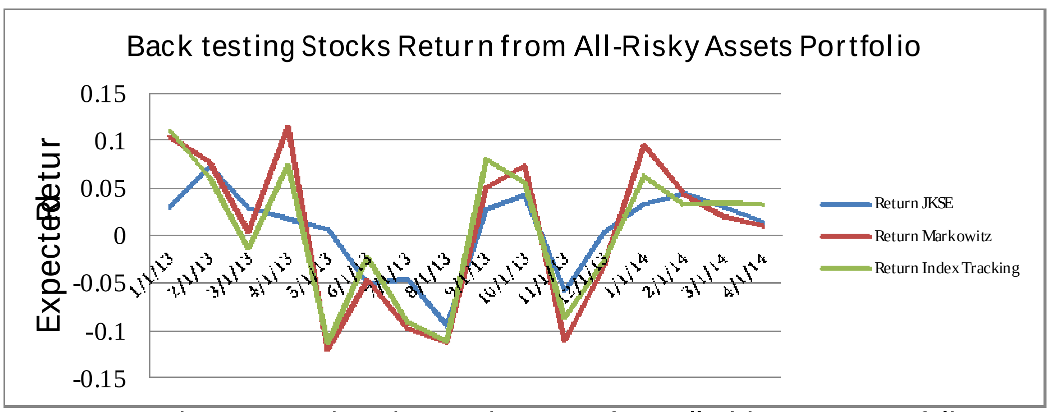

Figure 10: Backtesting Stocks Return from All-Risky Assets Portfolio

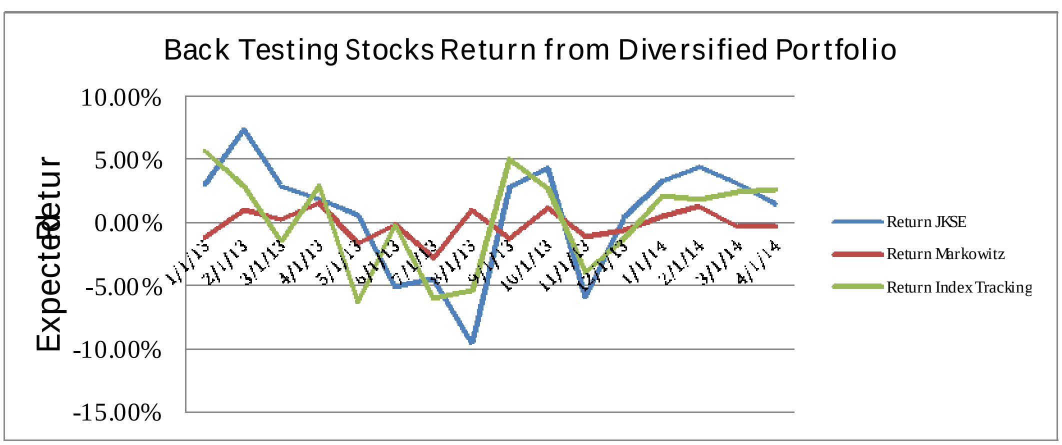

Figure 11: Backtesting Stocks Return from Diversified Portfolio

From the figures above, unlike the historical stock return performance figures, both the Markowitz MV model and index-tracking model do not track the benchmark index. This is due to the difference in the set of data used for different period, causing the change in the mean return, risk, and beta of the portfolio and the benchmark index. The index mean return of the new period is r_{M2} = 0.71536%. The difference of index-tracking and Markowitz MV portfolio performance in all-risky assets portfolio is not as significant as the performance of index-tracking and Markowitz MV portfolio during the new period. Table 3 below shows the risk, return, and Sharpe ratio of each stocks portfolio under the new period of observation.

	Mean Return	Risk	Sharpe Ratio
JKSE	0.7154%	4.5303%	0.02697
Index-Tracking Risky Assets	0.5860%	7.2959%	-0.00098
Index-Tracking with Risk-Free Assets	0.4010%	3.8755%	-0.04957
Markowitz Risky Assets	0.5370%	8.2268%	-0.00683
Markowitz with Risk-Free Assets	-0.4831%	1.2375%	-0.08895

Table 3: Stocks Portfolio Risk, Return, and Sharpe Ratio Under New Period

When the period has changed, no portfolio is able to beat the market. However, compare to portfolios which uses the Markowitz MV model, the portfolios with index-tracking method perform better. It is indicated by the higher values of Sharpe ratio. As shown from the table 3, the index-tracking risky assets portfolio has a Sharpe ratio of -0.00098 while the Markowitz MV risky assets portfolio has a Sharpe ratio of -0.00683. As for the index-tracking mixed portfolio the Sharpe ratio is -0.04957, while the Markowitz MV mixed portfolio has the Sharpe ratio of -0.08895. This means that although the Markowtiz portfolio with risk-free assets remains to be the portfolio with the lowest risk of s_{P7} = 1.2375%, the portfolio underperforms all other portfolios. The portfolio which gives the highest return is the index-tracking risky assets portfolio with r_{P5} = 0.5860%, while the portfolio with the highest risk is Markowitz MV risky assets portfolio with the standard deviation of s_{P6} = 8.2268%. The Markowitz portfolio with risk-free assets remains the portfolio with the lowest risk of s_{P7} = 1.2375%.

Portfolio Performance Comparison

The portfolio performance is further measured by using three methods, which are the Sharpe, Traynor, and Jensen's Alpha measurement. Based on the three measurements, table 4 represents the calculation of each portfolio performance.

Portfolio	Sharpe Ratio	Alpha	Treynor Measure
Index-Tracking Risky Assets	0.05412	-0.00210	0.00461
Index-Tracking with Risk-Free Assets	0.08765	0.00083	0.00714
Markowitz Risky Assets	0.05659	-0.00094	0.00536
Markowitz with Risk-Free Assets	0.24750	0.00536	0.04689

Table 4: Portfolio Performance

From the table above, the Markowitz MV portfolio with risk-free assets outperforms other portfolios. This happens because the standard deviation of the portfolio is really low under the same level of expected mean return. However, the main objective of this research is to generate a portfolio which can track the benchmark index, and the Markowitz MV portfolio with risk-free assets does not track the benchmark index.

Conclusions

After computing the portfolio for all-risky assets and the diversified assets using both Markowitz MV and index-tracking model, each portfolio results in different level of risk with the desired mean return of r_M = 1.211%. Additional risk-free asset lowers the risk significantly for both Markowitz and the index-tracking portfolios. The index-tracking portfolio risk with risk-free assets is even lower than the risk of the market. However, despite having the lower risk than the index-tracking portfolio, the Markowitz MV portfolio is not really well diversified. At the lower thresholds of risk, the index-tracking portfolio is more diversified than the Markowitz MV portfolio. The index-tracking portfolio also gives the higher beta than the Markowitz MV portfolio. This increase in beta depends on the index variance, in this case JKSE variance, and also the asset covariance matrix. During the backtesting, 20% period of data are used to measure the performance of the portfolio from a different set of time. As one can expect, the performance of both Markowitz MV portfolio and index-tracking portfolio do not track the index performance. Even the previously most tracked portfolio, the index-tracking portfolio with risk-free assets, does not really track the index barometer anymore. However, compared to Markowitz MV model, the index-tracking portfolio tracks the market better under the portfolio with risk-free assets addition.

The portfolio performance is further measured by using three methods, which are the Sharpe, Traynor, and Jensen's Alpha measurement. From the table above, the Markowitz MV portfolio with risk-free assets outperforms other portfolios. This happens because the standard deviation of the portfolio is really low under the same level of expected mean return. However, the main objective of this research is to generate a portfolio which can track the benchmark index, and the Markowitz MV portfolio with risk-free assets does not track the benchmark index. Finally, a portfolio can be chosen based on the needs of the investor. The main objective of this research is to construct the optimal portfolio which can track the benchmark index. In addition of the risk-free assets, the portfolio tracks best the benchmark index and also minimize the risk. Therefore, if the index-tracking model is to be used, the addition of risk-free assets on the portfolio can be one of the considerations to lower the risk which usually exists when using the index-tracking method.

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