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Trustworthiness in Collaboration: A Simple Hypergame with Relational Attitudes Analysis

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Abstract**—Trust is an important factor in negotiation and business collaboration and can be achieved through mutual understanding between characters involved in this social interaction. Only a few research using mathematical modeling treated the behavioral aspect of those characters, both rationally and emotionally. The purpose of this paper is to analyze and discuss the relationship between rational and emotional aspects of mutual understanding in a collaboration. This paper adopted and modified conventional concepts of misperception and psychological phenomena of the characters by using hypergame and attitudes analysis, respectively.**

First, I defined collaboration in social systems in term of normal form game-theoretical model, and then by using hypergame in which each character has their own internal model, I explained how rational understanding can be achieved. Next, using attitudes analysis, I discussed how important characters' attitude towards others are and showed that we need also a strong equilibrium of emotional understanding for a collaboration to produce trust as a result. Finally, this paper proposed a new concept of collaborative social systems equilibrium through such rigorous analysis.

Keywords**—Attitudes, Collaboration, Hypergame, Social Systems, Trust, Trustworthiness.**

INTRODUCTION

Trust is an important component in an interaction in everything we do, in our social system. Being trustworthy involves interacting with other characters' actions and believes in two aspects of trust: with rational action and intention. Rational action refers to strategies how you choose your action when interact with others, to follow through on what best for you and others surrounds you based on a rational model. Intention refers to emotional aspect behind those actions [7, 8], e.g., whether you have opportunistic or non-opportunistic behavior in your social interactions [6].

This paper showed and argued that to achieve trustworthiness in collaboration, both rational action, and emotional intention, which modeled with relational attitude analysis, have to be in the same strategy which resulted in agreement of both aspects. This is showed by analyzing whether an equilibrium is achieved or not from those interactions.

First, I explored the simple hypergames framework, and then explained how formal model of relational attitude in social systems [9, 10] can be enhanced and used. Next, I gave an example of social interaction to investigated the relationship between rational analysis modeled by hypergames and emotional analysis of attitudes model, then explained the findings. Finally, conclusions and further researches are discussed.

HYPERGAMES FRAMEWORK

Hypergame theory discusses the interactions of characters in social systems who may misunderstand some of the components of these interactions (which are modelled by game theory). This is the basic idea of hypergames where each character is assumed to have their own subjective view that, which is formulated as a normal form game called his subjective game, and decision making based on these. This way it allows the characters to have a different perception of the game.

Although hypergames have been developed in several ways [2-5], I focused on the simplest one, which is called simple hypergames and enhanced it by attitude analysis [9, 10], explained in Section III, to investigated trustworthiness in collaboration.

In this section, I explained games and hypergames as models of interaction in social systems. First, definitions of games in normal form [13], then hypergames, best response, dominant strategies, and finally formal model of attitudes will be discussed in Section IV.

Definition 1 (*Normal-form games*). A *game* G in normal form is a 3-tuple $\left(N, \left(S_p\right)_{p\in N}, \left(F_p\right)_{p\in N}\right)$, where $N = \{1,2,\dots,n\}$ is the set of all characters, S_p is the set of all strategies of character $p \in N$, and F_p is character p's preference on $S = \prod_{p\in N} S_p$. It is assumed that $|N| \geq 2$ and $\left|S_p\right| \geq 2$ for all $p \in N$. A component $s = \left(s_p\right)_{p\in N}$ in S is also defined as $\left(s_p, s_{-p}\right)$, where $s_{-p} = \left(s_q\right)_{q\neq p}$.

Definition 2 (*Best Responses* (BR)). For $p \in N$, $s_p \in S_p$, and $s^* \in S$, s_p is said to be a *best response* (BR) of character p at s^*, iff $\forall s_p' \in S_p, \left(s_p, s_{-p}^*\right) F_p \left(s_p', s_{-p}^*\right)$. The set of all BRs of p at s^* is denoted by $BR_p(s^*)$.

Definition 3 (*Dominant strategies* (DS)). For $p \in N$ and $s_p^* \in S_p$, s_p^* is said to a *dominant strategy* (DS) of character p, iff $\forall s \in S$, $s_p^* \in BR_p(s)$. DS_p denotes the set of all DSs of p.

Definition 4 (*Nash equilibrium* (NE)). For $s^* \in S$, s^* is said to be a *Nash equilibrium* (NE), iff $\forall p \in N$, $s_p^* \in BR_p(s^*)$. $NE(G)$ denotes the set of all NEs in a $G = \left(N, (S_p)_{p\in N}, (F_p)_{p\in N}\right)$.

Definition 5 (*Characters' Nash strategy*). A character's strategy that resulted in some Nash equilibrium (NE) is called *character's Nash strategy*, formally, $s_p^* \in S_p$ is called character p's Nash strategy in a normal-form game G iff there is exists $s_{-p} \in S_{-p}$ such that $(s_p^*, s_{-p}) \in NE(G)$. The set of all character p's Nash strategy in G is denoted by $NE_p(G)$.

Definition 6 (*Simple hypergames*) [1]. A *simple hypergame* $H = \left(N, (G^p)_{p\in N}\right)$ where $G^p = (N^p, S^p, F^p)$ is a normal form *game* called character p's subjective game. Set of characters N^p is the finite set of characters involved in G^p perceived by character p. It is assumed that $N^p \subseteq N$. Set of strategy $S^p = \prod_{q\in N^p} S_q^p$ perceived by character p, where S_q^p is the finite set of character q's strategies perceived by character p. Preference $F^p = \left(F_q^p\right)_{q\in N^p}$ is character q's preference perceived by character p, and $F_q^p : S^p \to \mathbb{R}$ for any $q \in N^p$.

Definition 7 (*Misperception in hypergames*) [1, 2, 3, 4, 5]. Character p has misperceived the set of characters involved in a situation iff $N^p \neq N$, some character q's set of strategy iff $S_q^p \neq S_q^q$, and some character q's preference iff $F_q^p \neq F_q^q$, with $p \neq q$. In hypergame, each character p chooses a strategy from S_p^p, $\prod_{p\in N} S_p^p$ is interpreted as the set of all outcomes from objective point of view.

In hypergames, each character is assumed to have his/her own subjective game of the interaction he/she believe he/she is involved in. A hypergame is a collection of those subjective games for each character. It is possible for characters involved to have a misperception regarding subjective game of other characters.

Definition 8 (*Hyper Nash equilibrium* (HNE)) [11, 12, 14]. Let $H = \left(N, (G^p)_{p\in N}\right)$ be a hypergame. Strategy $s^* = (s_p^*, s_{-p}^*) \in S^p$ is *hyper Nash equilibrium* of H iff $\forall p \in N$, $s_p^* \in NE_p(G^p)$. $HNE(H)$ denotes the set of all HNEs in H. By this definition, $HNE(H) = \prod_{p\in N} NE_p(G^p)$.

Note that, in hypergames, the set of all character q's Nash strategy perceived by character p in G^p is denoted by $NE_{qp}(G^p)$ and DS_{qp} denotes the set of all DSs of character q perceived by p, for all $p, q \in N$ and $p \neq q$. In a hyper Nash equilibrium, each character chooses a Nash strategy from his/her set of strategy, as long as every character has at least one $NE_p(G)$, $\forall p \in N$.

I used hypergames to model the rational interaction between characters which can have misperceptions. Next, I explained formal model of relational attitude in social systems.

FORMAL MODEL OF RELATIONAL ATTITUDE IN SOCIAL SYSTEMS

Based on [9, 10], this paper adopted and enhanced the concepts of attitudes and relational dominant strategy equilibrium into hypergames to analyzed characters' interactions.

Definition 9 (*Attitudes*). For $p \in N$, an element $a_p \in A_p$, is called attitude of character p, is a list $\left(a_{pq}\right)_{q\in N}$ of attitudes character p toward character q for each $q \in N$. Attitude $a_{pq} \in \{+, 0, -\}$ for $q \in N$. Positive, neutral, and negative attitude of character p toward character q are denoted as $a_{pq} = +$, $a_{pq} = 0$, and $a_{pq} = -$, respectively. A denotes $\prod_{p\in N} A_p$ and an element a of A is called an *attitude*. Notes that an attitude $a \in A$ is expressed with valued, directed, and complete graph with loops.

Definition 10 (*Social interactions*). A *social interaction SI* is a pair $\left(N, \left(A_p\right)_{p\in N}\right)$, where $N = \{1, 2, \ldots, n\}$ is the set of all characters and $\forall p \in N,\ A_p = \{+, -, 0\}^N$.

For $p \in N$, an element $a_p \in A_p$, called an attitude of character p, is a list $\left(a_{pq}\right)_{q\in N}$ of attitudes of character p toward character q for each $q \in N$, and $a_{pq} \in \{+, -, 0\}$ for $q \in N$. Character p attitudes toward character q in a social interaction are expressed by $a_{pq} = +$, $a_{pq} = -$, and $a_{pq} = 0$ for a positive, a negative, and a neutral attitude, respectively. A denotes $\prod_{p\in N} A_p$, and an element a of A is called an *attitude* (of a social interaction). A valued, directed graph with loop is used to represent an attitude $a \in A$ in a social interaction.

Definition 11 (*Social sytems*). A *social system SS* is a 3-tuple $\left(N, (G^p)_{p\in N}, \left(A_p\right)_{p\in N}\right)$, where $\left(N, (G^p)_{p\in N}\right)$ and $\left(N, \left(A_p\right)_{p\in N}\right)$ represent a simple hypergame H and a character's attitude in a social interaction SI, respectively. A pair $(s_p, a_p) \in S^p \times A_p$ of a strategy $s_p \in S^p$ and an attitude $a_p \in A_p$ of character $p \in N$ is called a *state* of character p in SS.

Definition 12 (*Accommodating response* (AR)). Let $G^p = (N^p, S^p, F^p)$ is character p's subjective game in hypergame H, for $p \in N$, $s_p \in S_p^p$, $s^* \in S^p$, and $q \in N$, s_p is said to be an *accommodating response* (AR) of character p at s^* to character q, if $(s_p, s_{-p}) F_q^p s^*$. $AR_q^p(s^*)$ denotes the set of all ARs of p at s^* to q in H.

Definition 13 (*Competing response* (CR)). Let $G^p = (N^p, S^p, F^p)$ is character p's subjective game in hypergame H, for $p \in N$, $s_p \in S_p^p$, $s^* \in S^p$, and $q \in N$, s_p is said to be an *competing response* (CR) of character p at s^* to character q, if $s^* F_q^p (s_p, s_{-p})$. $CR_q^p(s^*)$ denotes the set of all CRs of p at s^* to q in H.

Definition 14 (*Emotional response* (ER)). Let $G^p = (N^p, S^p, F^p)$ is character p's subjective game in hypergame H, for $p \in N$, $s_p \in S_p^p$, $s^* \in S^p$, $q \in N$, and $a^* \in A$, s_p is said to be an *emotional response* (ER) of character p at (s^*, a^*) to character q, if $[a_{pq}^* = + \, and \, s_p = AR_q^p(s^*)]$ or $[a_{pq}^* = - \, and \, s_p = CR_q^p(s^*)]$ or $[a_{pq}^* = 0]$. $ER_q^p(s^*, a^*)$ denotes the set of all ERs of p at (s^*, a^*) to q in H.

So, $\forall p, q \in N$, $\forall s^* \in S^p$, and $\forall a^* \in A$:

$$ER_q^p(s^*, a^*) = \begin{cases} AR_q^p(s^*) \; if \; a_{pq}^* = + \\ CR_q^p(s^*) \; if \; a_{pq}^* = - \\ S_p^p \qquad\quad if \; a_{pq}^* = 0 \end{cases}$$

Definition 15 (*Strategic emotional-response* (SER)). Let $G^p = (N^p, S^p, F^p)$ is character p's subjective game in hypergame H, for $p \in N$, $s_p \in S_p^p$, $s^* \in S^p$, $q \in N$, and $a^* \in A$, s_p is said to be a *strategic emotional-response* (SER) of character p at (s^*, a^*), if $\forall q \in N, s_p \in ER_q^p(s^*, a^*)$. The set of all SERs of character p at (s^*, a^*) in H are denoted by $SER_p(s^*, a^*)$.

Definition 16 (*Emotional dominant-strategy* (EDS)). Let $G^p = (N^p, S^p, F^p)$ is character p's subjective game in hypergame H, for $p \in N$, $s_p^* \in S^p$, and $a^* \in A$, s_p^* is said to be an *emotional dominant-strategy* (EDS) of character p at a^*, if $\forall s \in S^p, s_p^* \in SER_p(s, a^*)$. $EDS_p(a^*)$ denotes the set of all EDSs of character p at a^* in H.

Definition 17 (*Emotional dominant-strategy equilibrium* ($EDSE$)). Let $G^p = (N^p, S^p, F^p)$ is character p's subjective game in hypergame H, for $(s^*, a^*) \in S \times A$, is said to be an *emotional dominant-strategy equilibrium* ($EDSE$), if $\forall p \in N$, $s_p^* \in EDS_p(a^*)$. $EDSE(G^p)$ denotes the set of all emotional dominant-strategy equilibria in a subjective game G^p.

Definition 18 (*Joint-emotional equilibrium* (JEE)). Let $H = (N, (G^p)_{p \in N})$ be a hypergame and $G^p = (N^p, S^p, F^p)$ is character p's subjective game in hypergame H, for $(s^*, a^*) = ((s_p^*, s_{-p}^*), a^*) \in S \times A$, is said to be a *joint-emotional equilibrium* (JEE), if $\forall p \in N$, $s_p^* \in EDS_p(G^p, a^*)$. $JEE(H)$ denotes the set of all JEEs in hypergame H.

Definition 19 (*Collaborative social-systems equilibrium* ($CSSE$)). Let $H = (N, (G^p)_{p \in N})$, for $(s^*, a^*) \in S \times A$, $(s^*, a^*) = ((s_p^*, s_{-p}^*), a^*)$ is said to be a *collaborative social-systems equilibrium* ($CSSE$), if $\forall p \in N$, $s_p^* \in HNE(H)$ and $s_p^* \in JEE(H)$. The set off all $CSSE$s in a social system SS is denoted by $CSSE(SS)$.

Definition 20 (*Trustworthiness in collaboration*). Let $G^p = (N^p, S^p, F^p)$ is character p's subjective game in hypergame H, for all $p \in N$, if there is a collaborative social-system equilibrium ($CSSE(SS)$) then *trustworthiness in collaboration* for the interaction in a social system SS has been achieved.

Fig. 1 showed a model of two characters interaction. Character p and q are both having neutral attitude to him-/herself. Character p has negative attitude toward q, while q is positive to p.

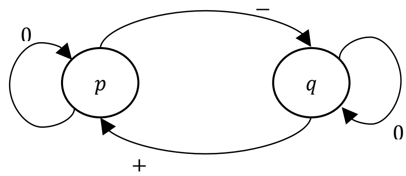

Fig. 1. An example of model of attitude in a social interaction between character p and character q using a valued, directed graph with loops.

IV. ANALYSIS OF SIMPLE HYPERGAMES WITH RELATIONAL ATTITUDES

In this section, I adopted those concepts of hypergames and attitudes analysis by using an example to analyze trust in a negotiation process of a business collaboration. As an example, consider a two-characters social interaction of p and q. Here, the social interaction is in form of negotiation process to collaborate between these characters.

I denote there are two characters involved in the negotiation as p and q, and represent the interaction by hypergame (G^p, G^q), see Table 1 and Table 2, respectively. Table 1 illustrates character p's subjective game, G^p, where $S_\mathrm{p}^p = \{p_1, p_2, p_3\}$, $S_q^p = \{q_1p, q_2p\}$ and each cell expresses preferences for character p and for character q perceived by p, i.e., F_p^p and F_q^p. Similarly, Table 2 represent character q's subjective game, G^q. In the hypergame both characters perceive the set of character involved correctly, i.e., $N^p = N^q = 2$, but misperceive each other's strategy set and preference.

Let me use Table 1 and Table 2 as an example of this misperception. In G^p, character p has three strategies, namely p_1, p_2, and p_3. Those strategies were percieved in character q's subjective game G^q as two strategies only, namely p_1q and p_2q. Meanwhile, character p perceived character q's strategies in G^p as q_1p and q_2p which have the same strategies as q_1 and q_2 in G^q, repectively, but with different preference ordering.

TABLE 1
CHARACTER p'S SUBJECTIVE GAME (G^p)

Strategy		S_q^p	
		q_1p	q_2p
	p_1	$3,1$	$4,4$
S_p^p	p_2	$2,2$	$5,5$
	p_3	$1,3$	$6,6$

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TABLE 2
CHARACTER q'S SUBJECTIVE GAME (G^q)

Strategy		S_q^q	
		q_1	q_2
S_p^q	p_1q	$1,1$	$2,3$
	p_2q	$3,2$	$4,4$

It is clear that each subjective game is a normal-form game. We can derive from Table 1 that in character p's subjective game G^p, $BR_p(G^p) = \{(p_1, q_1p), (p_3, q_2p)\}$, $BR_{qp}(G^p) = \{(p_1, q_2p), (p_2, q_2p), (p_3, q_2p)\}$ and $HNE(G^p) = (p_3, q_2p)$ since $NE_p(G^p) = \{p_3\}$ and $NE_{qp}(G^p) = \{q_2p\}$ are Nash strategies for character p and for character q perceived by p in G^p, respectively. In p's subjective game, $DS_p = \emptyset$ which means character p does not have dominant strategy, while $DS_{qp} = \{q_2p\}$.

With the same analysis from Table 2, in character q's subjective game G^q, $BR_{pq}(G^q) = \{(p_2q, q_1), (p_2q, q_2)\}$, $BR_q(G^q) = \{(p_1q, q_2), (p_2q, q_2)\}$ and $HNE(G^q) = (p_2q, q_2)$ since $NE_{pq}(G^q) = \{p_2q\}$ and $NE_q(G^q) = \{q_2\}$ are Nash strategies for character p perceived by q and for q in G^q, respectively. In q's subjective game, both characters have a dominant strategy, i.e., $DS_{pq} = \{p_2q\}$ and $DS_q = \{q_2\}$.

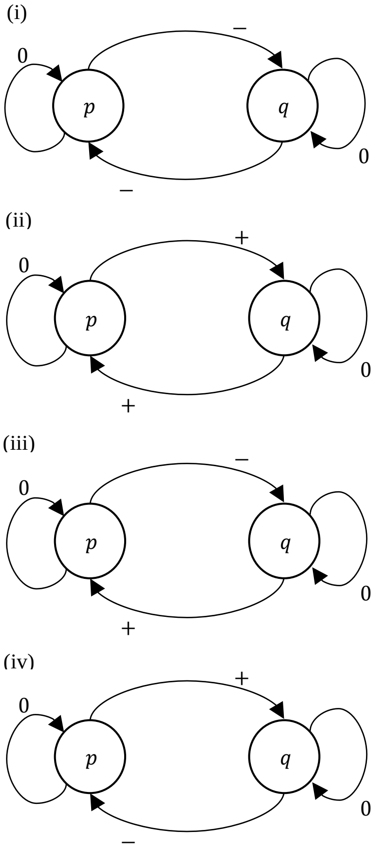

Fig. 2. Attitude graphs for interaction: (i) both characters p and q have opportunistic behavior; (ii) both characters p and q have non-opportunistic behavior; (iii) character p has opportunistic behavior and q has non-opportunistic one, and (iv) character p has non-opportunistic behavior and q has opportunistic one.

I distinguish characters in a social system into two types: opportunist and non-opportunist [6]. In this paper, I assume that a character has opportunistic behaviour if he/she has negative attitude toward others and has neutral attitude to him-/herself. On the other hand, I called the character has non-opportunistic behaviour if he/she has positive attitude toward others and has neutral attitude to him-/herself.

I analysed interaction of hypergame (G^p, G^q) with combination of character p and q's opportunistic or non-opportunistic behavior. We shall have four scenarios, i.e., (i) both characters p and q have opportunistic behavior; (ii) both characters p and q have non-opportunistic behavior; (iii) character p has opportunistic behavior and q has non-opportunistic one, and (iv) character p has non-opportunistic behavior and q has opportunistic one. The attitude graphs are illustrated in Fig. 2. In this paper, for all scenarios, I presume each character has neutral attitude toward him-/herself. Next, I investigated the relationship between hyper Nash equilibrium (HNE) and joint-emotional equilibrium (JEE) in each scenario.

Proposition Trustworthiness in collaboration can be achieved when all characters whose interacted in a social system have non-opportunistic behavior, i.e., positive attitude toward others.

SCENARIO I: HYPERGAME AND ATTITUDE ANALYSIS WHEN BOTH CHARACTERS p AND q HAVE OPPORTUNISTIC BEHAVIOR

In Scenario I, both characters have competing response (CR) to each other as their emotional response (ER). Table 3 shows emotional response in G^p which is based on Table 1, while Table 4 in G^q based on Table 2. Competing response in G^p means that character p will choose s_p which will give q an outcome indifferent or less preferable to s^*. Similar meaning for character q in G^q.

TABLE 3
$ER_q^p(s^*, a^*)$ IN G^p FOR SCENARIO I

	$(\boldsymbol{p_1, q_1p})$	$(\boldsymbol{p_2, q_1p})$	$(\boldsymbol{p_3, q_1p})$	$(\boldsymbol{p_1, q_2p})$	$(\boldsymbol{p_2, q_2p})$	$(\boldsymbol{p_3, q_2p})$
$\boldsymbol{ER_p^p}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$
$\boldsymbol{ER_q^p}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2, p_3\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2, p_3\}$
$\boldsymbol{ER_p^q}$	$\{q_1p\}$	$\{q_1p\}$	$\{q_1p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$
$\boldsymbol{ER_q^q}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$

TABLE 4
$ER_p^q(s^*, a^*)$ IN G^q FOR SCENARIO I

	$(\boldsymbol{p_1q, q_1})$	$(\boldsymbol{p_2q, q_1})$	$(\boldsymbol{p_1q, q_2})$	$(\boldsymbol{p_2q, q_2})$
$\boldsymbol{ER_p^p}$	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$
$\boldsymbol{ER_p^q}$	$\{p_1q\}$	$\{p_1q, p_2q\}$	$\{p_1q\}$	$\{p_1q, p_2q\}$
$\boldsymbol{ER_q^p}$	$\{q_1\}$	$\{q_1\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
$\boldsymbol{ER_q^q}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$

In Table 5, we see strategic emotional-response (SER) of character p and of character q perceived by p in G^p which is derived from Table 3. Table 6, derived from Table

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4, shows SER of character p perceived by q and of character q in G^q.

TABLE 5

$SER_p(s^*, a^*)$ AND $SER_{qp}(s^*, a^*)$ IN G^p FOR SCENARIO I

	(p_1, q_1p)	(p_2, q_1p)	(p_3, q_1p)	(p_1, q_2p)	(p_2, q_2p)	(p_3, q_2p)
SER_p	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2, p_3\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2, p_3\}$
SER_{qp}	$\{q_1p\}$	$\{q_1p\}$	$\{q_1p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$

TABLE 6

$SER_{pq}(s^*, a^*)$ AND $SER_q(s^*, a^*)$ IN G^q FOR SCENARIO I

	(p_1q, q_1)	(p_2q, q_1)	(p_1q, q_2)	(p_2p, q_2)
SER_{pq}	$\{p_1q\}$	$\{p_1q, p_2q\}$	$\{p_1q\}$	$\{p_1q, p_2q\}$
SER_q	$\{q_1\}$	$\{q_1\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$

After we knew all the set of strategic emotional-response for both characters in each subjective game, we can analyze to obtain whether any of the characters have emotional dominant-strategy (EDS) and look if we have emotional dominant-strategy equilibrium ($EDSE$) in this scenario.

The emotional dominant-strategies for character p and character q perceived by p in p's subjective game G^p are shown in Table 7. Meanwhile Table 8 shows for G^q. From those tables, we got $EDSE(G^p) = \{(p_1, q_1p), a^*\}$ and $EDSE(G^q) = \{(p_1q, q_1), a^*\}$.

TABLE 7

$EDS_p(a^*)$, $EDS_{qp}(a^*)$ AND $EDSE(G^p)$ IN G^p FOR SCENARIO I

$EDS_p(a^*) = \{p_1\}$

$EDS_{qp}(a^*) = \{q_1p\}$

$EDSE(G^p) = \{(p_1, q_1p), a^*\}$

TABLE 8

$EDS_{pq}(a^*)$, $EDS_q(a^*)$ AND $EDSE(G^q)$ IN G^q FOR SCENARIO I

$EDS_p(a^*) = \{p_1q\}$

$EDS_{qp}(a^*) = \{q_1\}$

$EDSE(G^q) = \{(p_1q, q_1), a^*\}$

Based on my previous analysis of Table 1 and Table 2, we knew that $NE(G^p) = (p_3, q_2p)$, $NE(G^q) = (p_2q, q_2)$ and $HNE(H) = HNE(G^p, G^q) = (p_3, q_2)$. It means that both character p and q rationally are are doing his-/her best strategy available which resulting in Nash equilibrium. In other words, they are doing their Nash strategy.

Meanwhile, from the attitude analysis, p and q emotionally have different strategies with their rational ones due to $EDSE(G^p) = \{(p_1, q_1p), a^*\}$, $EDSE(G^q) = \{(p_1q, q_1), a^*\}$ and $JEE(H) = JEE(G^p, G^q) = \{(p_1, q_1), a^*\}$. So, Scenario I do not result in a collaborative social systems equilibrium, $CSSE\ (SS) = \emptyset$, even though those strategies are resulting at equilibrium in their respective analysis. Based on previous definition, there is no trustworthiness in this collaboration.

Next, I showed the analysis if both characters have positive attitude towards the other.

SCENARIO II: HYPERGAME AND ATTITUDE ANALYSIS WHEN BOTH CHARACTERS p AND q HAVE NON-OPPORTUNISTIC BEHAVIOR

In Scenario II, both characters have accommodating response (AR) to each other as their emotional response (ER). Table 9 shows emotional response in G^p which is based on Table 1, while Table 10 in G^q based on Table 2. Accommodating response in G^p means that character p will choose s_p which will give q an outcome indifferent or more preferable to s^*. Similar meaning for character q in G^q.

TABLE 9

$ER_q^p(s^*, a^*)$ IN G^p FOR SCENARIO II

	(p_1, q_1p)	(p_2, q_1p)	(p_3, q_1p)	(p_1, q_2p)	(p_2, q_2p)	(p_3, q_2p)
ER_p^p	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$
ER_q^p	$\{p_1, p_2, p_3\}$	$\{p_2, p_3\}$	$\{p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_2, p_3\}$	$\{p_3\}$
ER_p^q	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_2p\}$	$\{q_2p\}$	$\{q_2p\}$
ER_q^q	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$

TABLE 10

$ER_p^q(s^*, a^*)$ IN G^q FOR SCENARIO II

	(p_1q, q_1)	(p_2q, q_1)	(p_1q, q_2)	(p_2q, q_2)
ER_p^p	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$
ER_p^q	$\{p_1q, p_2q\}$	$\{p_2q\}$	$\{p_1q, p_2q\}$	$\{p_2q\}$
ER_q^p	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$
ER_q^q	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$

In Table 11, we see strategic emotional-response (SER) of character p and of character q perceived by p in G^p which is derived from Table 9. Table 12, derived from Table 10, shows SER of character p perceived by q and of character q in G^q.

TABLE 11

$SER_p(s^*, a^*)$ AND $SER_{qp}(s^*, a^*)$ IN G^p FOR SCENARIO II

	(p_1, q_1p)	(p_2, q_1p)	(p_3, q_1p)	(p_1, q_2p)	(p_2, q_2p)	(p_3, q_2p)
SER_p	$\{p_1, p_2, p_3\}$	$\{p_2, p_3\}$	$\{p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_2, p_3\}$	$\{p_3\}$
SER_{qp}	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_2p\}$	$\{q_2p\}$	$\{q_2p\}$

TABLE 12

$SER_{pq}(s^*, a^*)$ AND $SER_q(s^*, a^*)$ IN G^q FOR SCENARIO II

	(p_1q, q_1)	(p_2q, q_1)	(p_1q, q_2)	(p_2p, q_2)
SER_{pq}	$\{p_1q, p_2q\}$	$\{p_2q\}$	$\{p_1q, p_2q\}$	$\{p_2q\}$
SER_q	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$

The emotional dominant-strategies for character p and character q perceived by p in p's subjective game G^p are shown in Table 13. Meanwhile Table 14 shows for G^q. From those tables, we got $EDSE(G^p) = \{(p_3, q_2p), a^*\}$ and $EDSE(G^q) = \{(p_2q, q_2), a^*\}$.

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TABLE 13
$EDS_p(a^*)$, $EDS_{qp}(a^*)$ AND $EDSE(G^p)$ IN G^p FOR SCENARIO II

$\boldsymbol{EDS_p(a^*) = \{p_3\}}$
$\boldsymbol{EDS_{qp}(a^*) = \{q_2p\}}$
$\boldsymbol{EDSE(G^p) = \{(p_3, q_2p), a^*\}}$

TABLE 14
$EDS_{pq}(a^*)$, $EDS_q(a^*)$ AND $EDSE(G^q)$ IN G^q FOR SCENARIO II

$\boldsymbol{EDS_p(a^*) = \{p_2q\}}$
$\boldsymbol{EDS_{qp}(a^*) = \{q_2\}}$
$\boldsymbol{EDSE(G^q) = \{(p_2q, q_2), a^*\}}$

We knew from hypergame analysis that $NE(G^p) = (p_3, q_2p)$, $NE(G^q) = (p_2q, q_2)$ and $HNE(H) = HNE(G^p, G^q) = (p_3, q_2)$. While from attitude analysis we got $EDSE(G^p) = \{(p_3, q_2p), a^*\}$, $EDSE(G^q) = \{(p_2q, q_2), a^*\}$ and $JEE(H) = JEE(G^p, G^q) = \{(p_3, q_2), a^*\}$. We see that in each respective subjective game of G^p and G^q, character p and q rationally and emotionally are choosing same strategy, i.e., strategy p_3 for p in G^p and q_2 for q in G^q. These strategies are resulted in equilibria.

We see that in Scenario II, we have a collaborative social systems equilibrium, i.e., $CSSE(SS) = CSSE(H) = \{(p_3, q_2)\}$. Moreover, based on Definition 20, we can say that in this interaction, character p and q have achieved trustworthiness in collaboration.

Lastly, I combined the analysis for Scenario III and Scenario IV because they have the similar situation in which one character has opportunistic behavior and the other do not.

SCENARIO III: HYPERGAME AND ATTITUDE ANALYSIS WHEN CHARACTER p HAS OPPORTUNISTIC BEHAVIOR WHILE CHARACTER q HAS NON-OPPORTUNISTIC ONE

Table 15 shows emotional response in G^p which is based on Table 1, while Table 16 in G^q based on Table 2. Accommodating response in G^pmeans that character p will choose s_p which will give q an outcome indifferent or more preferable to s^*. Similar meaning for character q in G^q.

TABLE 15
$ER_q^p(s^*, a^*)$ IN G^p FOR SCENARIO III

	(p_1, q_1p)	(p_2, q_1p)	(p_3, q_1p)	(p_1, q_2p)	(p_2, q_2p)	(p_3, q_2p)
ER_p^p	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$
ER_q^p	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2, p_3\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2, p_3\}$
ER_p^q	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_2p\}$	$\{q_2p\}$	$\{q_2p\}$
ER_q^q	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$

TABLE 16
$ER_p^q(s^*, a^*)$ IN G^q FOR SCENARIO III

	(p_1q, q_1)	(p_2q, q_1)	(p_1q, q_2)	(p_2q, q_2)
ER_p^p	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$	$\{p_1q, p_2q\}$
ER_p^q	$\{p_1q\}$	$\{p_1q, p_2q\}$	$\{p_1q\}$	$\{p_1q, p_2q\}$
ER_q^p	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$
ER_q^q	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$

In Table 17, we see strategic emotional-response (SER) of character p and of character q perceived by p in G^p which is derived from Table 15. Table 18, derived from Table 16, shows SER of character p perceived by q and of character q in G^q.

TABLE 17
$SER_p(s^*, a^*)$ AND $SER_{qp}(s^*, a^*)$ IN G^p FOR SCENARIO III

	(p_1, q_1p)	(p_2, q_1p)	(p_3, q_1p)	(p_1, q_2p)	(p_2, q_2p)	(p_3, q_2p)
SER_p	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2, p_3\}$	$\{p_1\}$	$\{p_1, p_2\}$	$\{p_1, p_2, p_3\}$
SER_{qp}	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_1p, q_2p\}$	$\{q_2p\}$	$\{q_2p\}$	$\{q_2p\}$

TABLE 18
$SER_{pq}(s^*, a^*)$ AND $SER_q(s^*, a^*)$ IN G^q FOR SCENARIO III

	(p_1q, q_1)	(p_2q, q_1)	(p_1q, q_2)	(p_2p, q_2)
SER_{pq}	$\{p_1q\}$	$\{p_1q, p_2q\}$	$\{p_1q\}$	$\{p_1q, p_2q\}$
SER_q	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$	$\{q_2\}$

The emotional dominant-strategies for character p and character q perceived by p in p's subjective game G^p are shown in Table 19. Meanwhile Table 20 shows for G^q. From those tables, we got $EDSE(G^p) = \{(p_1, q_2p), a^*\}$ and $EDSE(G^q) = \{(p_1q, q_2), a^*\}$.

TABLE 19
$EDS_p(a^*)$, $EDS_{qp}(a^*)$ AND $EDSE(G^p)$ IN G^p FOR SCENARIO III

$\boldsymbol{EDS_p(a^*) = \{p_1\}}$
$\boldsymbol{EDS_{qp}(a^*) = \{q_2p\}}$
$\boldsymbol{EDSE(G^p) = \{(p_1, q_2p), a^*\}}$

TABLE 20
$EDS_{pq}(a^*)$, $EDS_q(a^*)$ AND $EDSE(G^q)$ IN G^q FOR SCENARIO III

$\boldsymbol{EDS_p(a^*) = \{p_1q\}}$
$\boldsymbol{EDS_{qp}(a^*) = \{q_2\}}$
$\boldsymbol{EDSE(G^q) = \{(p_1q, q_2), a^*\}}$

From Table 1 and Table 2, we knew that $NE(G^p) = (p_3, q_2p)$, $NE(G^q) = (p_2q, q_2)$ and $HNE(H) = HNE(G^p, G^q) = (p_3, q_2)$. While from attitude analysis we got $EDSE(G^p) = \{(p_1, q_2p), a^*\}$, $EDSE(G^q) = \{(p_1q, q_2), a^*\}$ and $JEE(H) = JEE(G^p, G^q) = \{(p_1, q_2), a^*\}$. Scenario III do not result in a collaborative social system equilibrium, in other word, $CSSE\ (SS) = \emptyset$. And because of this, there is no trustworthiness in this collaboration.

Same analysis could be done in a similar manner for Scenario IV.

SCENARIO IV: HYPERGAME AND ATTITUDE ANALYSIS WHEN CHARACTER p HAS NON-OPPORTUNISTIC BEHAVIOR WHILE CHARACTER q HAS OPPORTUNISTIC ONE

Table 21 shows emotional response in G^p which is based on Table 1, while Table 22 in G^q based on Table 2.

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Accommodating response in G^pmeans that character p will choose s_p which will give q an outcome indifferent or more preferable to s^*. Similar meaning for character q in G^q.

TABLE 21
$ER_q^p(s^*, a^*)$ IN G^p FOR SCENARIO IV

	$(\boldsymbol{p_1}, \boldsymbol{q_1 p})$	$(\boldsymbol{p_2}, \boldsymbol{q_1 p})$	$(\boldsymbol{p_3}, \boldsymbol{q_1 p})$	$(\boldsymbol{p_1}, \boldsymbol{q_2 p})$	$(\boldsymbol{p_2}, \boldsymbol{q_2 p})$	$(\boldsymbol{p_3}, \boldsymbol{q_2 p})$
$\boldsymbol{ER_p^p}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_1, p_2, p_3\}$
$\boldsymbol{ER_q^p}$	$\{p_1, p_2, p_3\}$	$\{p_2, p_3\}$	$\{p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_2, p_3\}$	$\{p_3\}$
$\boldsymbol{ER_p^q}$	$\{q_1 p\}$	$\{q_1 p\}$	$\{q_1 p\}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$
$\boldsymbol{ER_q^q}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$

TABLE 22
$ER_p^q(s^*, a^*)$ IN G^q FOR SCENARIO IV

	$(\boldsymbol{p_1 q}, \boldsymbol{q_1})$	$(\boldsymbol{p_2 q}, \boldsymbol{q_1})$	$(\boldsymbol{p_1 q}, \boldsymbol{q_2})$	$(\boldsymbol{p_2 q}, \boldsymbol{q_2})$
$\boldsymbol{ER_p^p}$	$\{p_1 q, p_2 q\}$	$\{p_1 q, p_2 q\}$	$\{p_1 q, p_2 q\}$	$\{p_1 q, p_2 q\}$
$\boldsymbol{ER_p^q}$	$\{p_1 q, p_2 q\}$	$\{p_2 q\}$	$\{p_1 q, p_2 q\}$	$\{p_2 q\}$
$\boldsymbol{ER_q^p}$	$\{q_1\}$	$\{q_1\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$
$\boldsymbol{ER_q^q}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$

In Table 23, we see strategic emotional-response (SER) of character p and of character q perceived by p in G^p which is derived from Table 21. Table 24, derived from Table 22, shows SER of character p perceived by q and of character q in G^q.

TABLE 23
$SER_p(s^*, a^*)$ AND $SER_{qp}(s^*, a^*)$ IN G^p FOR SCENARIO IV

	$(\boldsymbol{p_1}, \boldsymbol{q_1 p})$	$(\boldsymbol{p_2}, \boldsymbol{q_1 p})$	$(\boldsymbol{p_3}, \boldsymbol{q_1 p})$	$(\boldsymbol{p_1}, \boldsymbol{q_2 p})$	$(\boldsymbol{p_2}, \boldsymbol{q_2 p})$	$(\boldsymbol{p_3}, \boldsymbol{q_2 p})$
$\boldsymbol{SER_p}$	$\{p_1, p_2, p_3\}$	$\{p_2, p_3\}$	$\{p_3\}$	$\{p_1, p_2, p_3\}$	$\{p_2, p_3\}$	$\{p_3\}$
$\boldsymbol{SER_{qp}}$	$\{q_1 p\}$	$\{q_1 p\}$	$\{q_1 p\}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$	$\{q_1 p, q_2 p\}$

TABLE 24
$SER_{pq}(s^*, a^*)$ AND $SER_q(s^*, a^*)$ IN G^q FOR SCENARIO IV

	$(\boldsymbol{p_1 q}, \boldsymbol{q_1})$	$(\boldsymbol{p_2 q}, \boldsymbol{q_1})$	$(\boldsymbol{p_1 q}, \boldsymbol{q_2})$	$(\boldsymbol{p_2 p}, \boldsymbol{q_2})$
$\boldsymbol{SER_{pq}}$	$\{p_1 q, p_2 q\}$	$\{p_2 q\}$	$\{p_1 q, p_2 q\}$	$\{p_2 q\}$
$\boldsymbol{SER_q}$	$\{q_1\}$	$\{q_1\}$	$\{q_1, q_2\}$	$\{q_1, q_2\}$

The emotional dominant-strategies for character p and character q perceived by p in p's subjective game G^p are shown in Table 25. Meanwhile Table 26 shows for G^q. From those tables, we got $EDSE(G^p) = \{(p_3, q_1 p), a^*\}$ and $EDSE(G^q) = \{(p_2 q, q_1), a^*\}$.

TABLE 25
$EDS_p(a^*)$, $EDS_{qp}(a^*)$ AND $EDSE(G^p)$ IN G^p FOR SCENARIO IV

$\boldsymbol{EDS_p(a^*)} = \{\boldsymbol{p_3}\}$
$\boldsymbol{EDS_{qp}(a^*)} = \{\boldsymbol{q_1 p}\}$
$\boldsymbol{EDSE(G^p)} = \{(\boldsymbol{p_3}, \boldsymbol{q_1 p}), \boldsymbol{a^*}\}$

TABLE 26
$EDS_{pq}(a^*)$, $EDS_q(a^*)$ AND $EDSE(G^q)$ IN G^q FOR SCENARIO IV

$\boldsymbol{EDS_p(a^*)} = \{\boldsymbol{p_2 q}\}$
$\boldsymbol{EDS_{qp}(a^*)} = \{\boldsymbol{q_1}\}$
$\boldsymbol{EDSE(G^q)} = \{(\boldsymbol{p_2 q}, \boldsymbol{q_1}), \boldsymbol{a^*}\}$

Similar like other scenarios, we knew from Table 1 and Table 2 that $NE(G^p) = (p_3, q_2 p)$, $NE(G^q) = (p_2 q, q_2)$ and $HNE(H) = HNE(G^p, G^q) = (p_3, q_2)$. While from attitude analysis we got $EDSE(G^p) = \{(p_3, q_1 p), a^*\}$, $EDSE(G^q) = \{(p_2 q, q_1), a^*\}$ and $JEE(H) = JEE(G^p, G^q) = \{(p_3, q_1), a^*\}$. Like Scenario I and Scenario III, Scenario IV also do not result in a collaborative social system equilibrium, in other word, $CSSE\ (SS) = \emptyset$. And because of this, there is no trustworthiness in this collaboration.

Based on analyses of these scenarios, we see that only Scenario II in which both characters have non-opportunistic behaviors in the interaction resulted in *trustworthiness in collaboration* situation. This result implies that analyzing trust in social interactions needs to be seen from two aspects, rational and emotional.

V. CONCLUSION AND FURTHER RESEARCH

This paper enhanced the concept of attitude analysis into hypergames and investigated their relationship with trust in collaboration. I first used hypergames as a framework for rational analysis and formal model of attitude as emotional analysis. Then by combining both to analyze the relationship between rational and emotional aspects, I showed how trustworthiness in collaboration can be achieved. Next, I analyze four scenarios of oppotunistic and non-opportunistic behavior of characters in a social interaction as an example. Finally, I discussed the findings of those scenarios.

This research is a beginning phase to understand how trust developed in a collaboration by using static analysis. Next, it is also important to find out how trust developed over time in a dynamic situation.

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