

Recent Advances in Real Options Signaling Games Literature

George Rigopoulos

Department of Informatics, Technological Educational Institute (TEI) of Athens, Athens, Greece

Abstract: Financial economics have benefited from game theory advances and its application to various domains where competition is present. Following relevant research this paper presents a literature review of recent works on the intersection of real options and game theory, especially on signaling games. A preliminary background is presented initially and next a review of the works is presented. The paper compiles existing works to a useful reference for researchers in the field.

Keywords: Signaling games; Capital budgeting, Real options, Game theory, Discrete time games

1. Introduction

Real world investments do not happen in an isolated environment, but to complex settings where competition is present and very demanding in terms of strategic decisions. Game theory is the par excellence methodological tool for coping with competition and has been used to analyze simple or more complex cases. On the other hand, investments within capital budgeting domain have much benefited from the real option theory, which introduces the novel view of the delay option to an investment as another value point in addition to the expected cash flow. Both real option theory and game theory have evolved during the previous years independently, but researchers soon realized that their combination can offer a richer set of tools for analysis of even more complex settings. Thus, a number of works started to come out which offered models of real options combined with game theoretical models and a new domain was

actually generated. Nowadays the cross domain of real option theory and game theory is expanding and research interest is active as new tools and methodologies evolve to let for the solution of even more complex settings. The number and quality of publications is quite indicative of the research activity. Smit and Trigeorgis' book stands as the basic reference for real options terminology and background (Smit and Trigeorgis, 2004).

One of the less studied sub-domains is the area of games with incomplete information in combination with real options in strategic investments, or specifically real options signaling games. Existing literature is not quite extended in this field and there exist few recent approaches which, however, indicate the interest in the field on the one hand and its potential for research, and the complexity of solving such models on the other.

*Corresponding author. Email: grigop@teiath.gr

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In the following we present a brief analysis of research approaches in real options signaling games referencing all relevant academic publications. The scope of the survey is limited to works that combine signaling games and real options in order to provide useful background to the specific research stream. The aim of this work is to compile recent literature in a way to assist researchers in the field and not to provide all the relevant theoretical models in details. Interested reader should follow the references provided for background and works.

2. Overview

In the following we introduce a typical real option game model. We assume a duopoly setting where two firms are competing for the same investment opportunity. Two (identical for simplicity) firms exist in the market and compete for the same investment opportunity and each one holds an option to invest in the same project. The project infers an investment cost I , the investment is considered as sunk cost and generates an uncertain cash flow stream. In this setting the context is strategic as one's decision affects other's payoff, thus the need is to infer the optimal strategy.

In some cases, the investment may be unique and only one can benefit from it, so timing is crucial as the investment is available only for the first firm. So, the timing of option exercise is important as there is incentive to exercise first as soon as the payoff is positive. This results to a leader/follower competition situation, where the firm that invests first becomes the leader (or monopolist in case no further investment is allowed by the other firm) and the other becomes the follower if the other firm can invest in a second step. Consequently, the two firms either share the profits of the duopoly market as leader/follower or the leader get monopolist's profits. In details, the project generates output at a unit price P_t which is stochastically fluctuating over time according to the $P_t = X_t D[Q_t]$ where $D[Q_t]$ is the inverse demand function which is assumed to be downward sloping, X_t is a stochastic

exogenous shock process to demand which evolves as a Geometric Brownian Motion by $dX_t = \mu_x X_t dt + \sigma_x X_t dz$, and Q_t is the supply process. The firms in this setting have two choices, either to become leaders or followers, by selecting the investment timing accordingly. Solution process to the above is further presented by Azevedo and Paxson (Azevedo and Paxson, 2014). Assuming that the market positions (leader/follower) have already been defined by some process (either by chance or deterministically), then by utilizing backwards dynamic programming we can solve for the value functions. If the follower's value function is $F_F(X_t)$, under the assumption of risk neutral firms, it must solve for the following :

$$\frac{1}{2} \sigma_x^2 X_t^2 \frac{\partial^2 F_F(X_t)}{\partial X_t^2} + \mu_x \frac{\partial F_F(X_t)}{\partial X_t} - r F_F(X_t) = 0$$

with the following two boundary conditions that ensure the optimal strategy from the

follower $F_F(X_F^*) = \frac{X_F^* D(2)}{r - \mu_x} - I$, which is

called value matching condition and states the expected net present value of the duopoly payoff when the follower exercises its option,

and $F_F'(X_F^*) = \frac{D(2)}{r - \mu_x}$, which is called the

smooth pasting condition and ensures that the exercise trigger is chosen to maximize the option value where $D(2)$ is the output when both firms have invested and X_F^* is the follower's investment threshold (Azevedo & Paxson, 2014).

Following the real options theory the follower's optimal strategy is to exercise the option when $X_t > X_F^*$. The closed form solutions for value functions and investment thresholds for the follower and for the leader are found next (Azevedo & Paxson, 2014). The above setting is considered as a standard framework for real option games in the relevant literature. Probably one of the most challenging approaches in real option games is the case of real option games of incomplete information. This is the case where information asymmetry exists between

the players and there is also a real option value, for example a delay option to an investment. Signaling games comprise a special category of games with incomplete information and their theoretical setting is presented in more details in the following chapter. In the domain of real option games, the specific category is present in literature in a quite few number of publications. The reason is the inherent complexity which does not allow for analytical solutions of the models. The difficulty in such models lies in the fact that stochastic variables are included in the model. Thus there is an infinite amount of future paths and it is not possible to solve the model and provide with equilibrium results. In the majority of the relevant publications the equilibrium results are presented in formulas that provide a kind of entrance threshold for the investment. This issue is mentioned and explained by various authors (Watanabe, 2012). Specific equilibrium and corresponding payoffs are presented in very few works, including the present dissertation, under the application of certain assumptions that reduce the infinite values to a finite level. Despite the complexity, the fact that there exist recent publications on the field proves the increasing interest and research importance for the domain.

The publications on the field are presented in details in the following where we separate the models in continuous time models and models of discrete time. We consider games that there exists incomplete information between players, which means that some players have private information over the rest ones. Asymmetry in other game parameters may exist, for example in the players, but the information asymmetry or imperfect information leads to settings that are most challenging and relevant to the present research. From the existing publications, significant contributions to the subject comprise Grenadier's and Watanabe's work as they focus on real options signaling games in continuous setting, and van de Walle's one that utilizes discrete time setting (Grenadier, 1999; Grenadier 2000a; Smit and Trigeorgis, 2004; Van de Walle, 2012).

2. Continuous Time Models

A significant research stream is initiated by the work of Grenadier who presents in a number of publications a thorough study on either reviewing works on real options game models or introducing his own models on the subject. Grenadier in his early work introduces a real options game in duopoly market, which is further developed later in a signaling game after relaxing the assumptions and restrictions on the information and introducing information asymmetry and signaling between the players. In subsequent work he introduces more than two players in the model in a continuous time framework where he examines a firm's symmetric Nash equilibrium conditional on that the rest competitors follow their equilibrium strategies (Grenadier, 1996; 1999).

This article provides a general and tractable solution approach for deriving the equilibrium investment strategies of firms in a Cournot–Nash framework, while Grenadier claims that the resulting equilibrium is analytically simple and has potentially wide applications. In a recent work, which is a significant contribution to the field, Grenadier with Malenko present a real options signaling game formulation in a more concrete way, in which the decision to exercise an option is considered as a signal of private information to outsiders, whose beliefs affect the utility of the decision-maker.

The signaling incentives distort the timing of exercise, and the direction of distortion depends on whether the decision-maker's utility increases or decreases in outsiders' belief about the payoff from exercise. In the former case, signaling incentives erode the value of the option to wait and speed up option exercise, while in the latter case option exercise is delayed. Authors demonstrate the model's implications through four corporate finance settings: investment under managerial myopia, venture capital grandstanding, investment under cash flow diversion, and product market competition (Grenadier and Malenko, 2011).

Initially they begin with a general model of option exercise under asymmetric information. They consider a setting of outsiders and decision makers, where outsiders learn information about the decision maker from observing the exercise (or lack of exercise) of the option, and thereby change their assessment of the decision maker. Following that, and since the decision maker is aware of this information, the option exercise strategy is shaped to take advantage of it.

The payoff from the option exercise is comprised of two components; a fraction of the project's pay and the belief component, which depends on outsiders' assessment of the decision makers' type. The decision maker's type determines the project's NPV and is the private information of the decision maker. Authors focus on separating equilibrium in which the decision maker reveals her type through the option exercise strategy. They characterize a separating equilibrium of the general model, and prove that under standard regularity conditions it exists and is unique. Grenadier's findings include among other that the effect of information asymmetry on investment timing is not straightforward as information asymmetry can either speed up or delay investment, leading to overinvestment or underinvestment, respectively. Except these, Grenadier has also published literature review papers on game-theoretic option models and the way the intersection of real options and game theory provides insights into the behavior of economic agents under uncertainty and asymmetry of information (Grenadier, 2000b; 2002).

In comparison to Grenadier's work, real options models with imperfect information game setting are found in other publications, namely, Lambrecht and Perraudin (Lambrecht and Perraudin, 2003), and Hsu and Lambrecht (Hsu and Lambrecht, 2007). However, although they study options exercise games with information imperfections they conclude to very different equilibrium structures than Grenadier's latest paper (Grenadier, 2002). As commented by

Grenadier, each firm has an imperfect private signal about the true project value, while in Lambrecht and Perraudin each firm knows its own investment cost but not the investment cost of the competitor and in Hsu and Lambrecht, an incumbent is uninformed about the challenger's investment cost.

One characteristic of these works is that although they study options exercise with information imperfections of various forms, the beliefs of outsiders do not enter the payoff function of agents. Therefore, they are not pure examples of real options signaling games as the more informed player has no incentives to manipulate investment timing in a way to alter the belief of less informed players. On the other hand Morellec and Schurhoff (Morellec and Schurhoff, 2011) and Bustamante present models that are pure real options signaling games, close to Grenadier's work and thus can be included of in the context of our review. In Morellec and Schurhoff an informed firm, seeking external resources to finance an investment project, can select both the timing of investment and the means of financing (debt or equity) of the project. In Bustamante an informed firm can decide on both the timing of investment and whether to finance its investment project through an IPO or costlier private capital.

Bustamante and Morellec and Schurhoff find that asymmetric information speeds up investment as the firm attempts to signal better quality and thereby secure cheaper financing. Finally, Benmelech, Kandel, and Veronesi (Azevedo and Paxson, 2014) present a dynamic model of investment with asymmetric information between the manager and outsiders and show that in the presence of stock-based compensation, asymmetric information creates incentives to conceal bad news about growth options.

Another stream of research in real options game models comes from Imai and Watanabe who presented two working papers which focus on the investigation of managerial flexibility, modeled by real options, and competition, modeled by game theory (Junichi and Watanabe, 2005).

In their first paper Imai and Watanabe present a two stage investment game and develop a model of two firms competing in a leader-follower setting. The game is relative simple and the concept is to investigate the value of managerial flexibility in competition setting using a simple demand function which follows a one-period binomial process. The study is expanded in a multi stage setting in their second paper and additional concepts are included. The overall aim is to prove that the real option, as flexibility, adds value to the company; however results are not clear for the real option value in the case of flexibility existence in comparison to the case that it is not present.

Although the two papers seem to be not mature versions of authors' work and they do not focus on signaling, they provide some useful concepts on the modeling of a single stage binomial real options game. Moreover, they influenced, in a way, Watanabe's later papers in real options game with signaling (Watanabe, 2012). Watanabe in his recent works presents a two firm signaling game with real options in continuous setting. The two firms have asymmetric information and their revenue depend on the market structure, monopoly or duopoly, and the stochastic demand evolvement. The firms are considered as incumbent and entrant to the market, where the incumbent is assumed to get higher expected profit and invests earlier than the entrant.

This is considered as signaling behavior which affects the entrant's investment decision. The analysis is quite extended for the cases of high and low demand, where author reaches to conclusions about incumbent's optimal investment strategy. The profit flows involve two uncertain factors: (1) the basic level of the demand of the market observed only by the incumbent and (2) the fluctuation of the profit flow described by a geometric Brownian motion that is common to both firms. The optimal timing for the incumbent, who privately knows the high demand, is earlier than that for the low-demand incumbent. This earlier entrance, however, reveals the information of the high

demand to the entrant, so that the entrant observing the timing of the incumbent would accelerate its own timing of the investment that reduces the monopolistic profit of the incumbent. Therefore, the high-demand incumbent may delay the timing of the investment in order to hide the information strategically. The equilibriums of this signaling game are characterized, and the conditions for the manipulative revelation are investigated. The values of both firms are compared with the case of complete information. Although novel, Watanabe is following the stream from Grenadier's previous work on signaling games with real options (Décamps and Mariotti, 2004; Grenadier, 1996).

Another relevant work is the one of Décamps and Mariotti who present a model of asymmetric information between firms in a duopoly model of investment. They develop a model where the two firms have symmetric information about the value of the investment project, but asymmetric information about their investment costs. The investment project may be of low or high quality and they study the learning externality due to the increase in the signal's quality generated by the leader's investment. They consider investment costs as private information, and form the background signal as a Poisson process conditional on the quality of the project being low. The resulting attrition game has a unique, symmetric equilibrium which depends on initial public beliefs. If the project is of low quality, then players eventually learn this by observing failures that occur according to a Poisson process while a high-quality project never fails (Cobb and Basuchoudhary, 2009; Décamps et al., 2002).

Another relevant work comes from Zhu who presents in his works a model of competition under information asymmetry to study the strategic exercise of real options (Zhu, 1999). He approaches the study in a step by step basis, developing initially a multi period duopoly game between two firms under perfect information and examines how competition affect the early exercise of

options and aggressive investment behaviors and how competition erodes option values. Then he relaxes the symmetric information assumption and works the model with information asymmetry to exist between firms. His findings among others include that payoff is independent of the ordering of exercise, that the sequential exercise of real options may generate both informational and payoff externalities and that having more information is not necessarily better.

Zhu uses information asymmetry in the cost of firms, where one firm knows both cost while the other does not. The model is analyzed and presented in an extensive form tree for the basic instance of two periods and afterwards there is a generalization for the multi period model. The game is solved for the simple two period option and at the end of second period the payoffs are calculated for the final node. Then using a backward induction process Zhu calculates the optimal strategies for both firms. The payoffs are based on Cournot, monopoly or Stackelberg depending on the type of the equilibrium the game ends. Although Zhu uses the signaling in his game model, the idea that he adopts for the solution is not the perfect Bayesian equilibrium. He considers that at the end of the game the information asymmetry is mitigated as the more informed firm reveals its private cost information through its actions. Thus he considers that the payoff function can be calculated not by using the posterior beliefs but the prior beliefs. Also this concept allows for the use of backward induction till the initial node.

Zhu and Weyant present in 2003 a publication where they approach investment projects in new technologies as real options and study the effects of endogenous competition and asymmetric information on the strategic exercise of real options. They develop a multi-period, game-theoretic model and show how competition leads to early exercise and aggressive investment behaviors and how competition erodes option values. Then they relax the typical full-information assumption and allow information asymmetry to exist across firms. The model

shows that payoff is independent of the ordering of exercise, that the sequential exercise of real options may generate both informational and payoff externalities and that having more information is not necessarily better (Zhu and Weyant, 2003).

Kort in 2012 presented a work which builds upon Pawlina and Kort and involves incomplete information over investment costs only (Kort, 2012). This paper studies what happens to the value of waiting when a firm has incomplete information about its competitor in an oligopoly framework. They find that the value of waiting depends on the quality of the information, the size of the first-mover advantage and the number of competitors. They show that a unique and symmetric equilibrium exists under common and consistent priors. Investment is accelerated when one firm acquires complete information about the probability of the other firm, while the other firm's information remains incomplete. They calculate the value of (acquiring) information under different assumptions on the priors.

3. Discrete Time Models

The previous works present models in continuous time while the following is focusing on discrete setting. Thus, a binomial lattice approach to solve a real option investment game with signaling is presented by van de Walle (Van de Walle, 2012). Van de Walle approximates a real option investment game with asymmetric information by using binomial lattice model. The game consists of an incumbent having the information advantage, and an entrant who lacks information about a specific parameter, the investment costs. The investment decision depends on the present value of the project, the binomial parameters, the market share, the continuous-time discount rate and of course the private information which are the investment costs. A firm will undertake an investment if he expects his payoff to be positive and his timing of investment is a result of the possible option value of waiting. The main goal of this research is to show the influence

of asymmetric information (the information value) as well as possible option values of waiting (the option value) on the behavior of the firms, and ultimately demonstrate the combination of both and their superadditive value to the payoffs of the firm having the information advantage position.

Van de Walle formulates the game following the theoretical concepts of Grenadier and Watanabe, but he solves it applying a binomial lattice approach rather than a continuous one as followed by Grenadier and Watanabe. He works his solution on a basic two period model and extends the idea to a three period one. Although he expands his idea to a multi period game he does not generalize the solution as the complexity of the multi period model cannot be handled by his software approach (spreadsheet programming). The game is consisted of two players, which are firms with information asymmetry who compete for a project and share the profits of it. The profits of the firms are a share of the total present value of the project which is considered as stochastic variable that follows geometric Brownian motion. The game is formulated on the basis of signaling games and the solution is a weak perfect Bayesian equilibrium, according to the assumptions he declares.

After applying the solution he runs a set of simulations modifying various parameters and presents findings in a qualitative rather quantitative way. Limitations of his work are the complexity of the game in the case of more than three periods, the two players setting, and the value function which is a plain present value. In overall, van de Walle presents an approach which is more applied rather than theoretical and tries to limit the complexity of signaling games with real options.

4. Conclusion

As a conclusion we can identify that existing literature is not quite extended in this field and there exist few recent approaches where the majority of them focuses on real time models. However, literature indicates the

interest in the field on the one hand and its potential for research, and the complexity of solving such models on the other. Another point that should be mentioned is that although models have been improved in the past ten years, empirical evidence is not so extended so as to verify theoretical results and justify the validity of models.

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